Chapter 4A

Estimation and Testing

Introduction to Statistical Inference Methods

- Statistical Inference: Drawing conclusions about a population from sample data.

- Methods
  - Point Estimation – Using a sample statistic to estimate a parameter
  - Confidence Intervals – supplements an estimate of a parameter with an indication of its variability
  - Hypothesis Tests– assesses evidence for a claim about a parameter by comparing it with observed data

- Because a different sample might lead to different conclusions, we cannot be certain that our conclusions are 100% correct.
  - Statistical inference uses the language of probability to say how reliable our conclusions are.

Some Point Estimates

Parameter	Measure	Statistic
μ	Mean of a single population	$\bar{X}$
σ^2	Variance of a single population	S^2
σ	Standard deviation of a single population	S
p	Proportion of a single population	$\hat{p}$
$\mu_1 - \mu_2$	Difference in means of two populations	$\bar{X}_1 - \bar{X}_2$
$p_1 - p_2$	Difference in proportions of two populations	$\hat{p}_1 - \hat{p}_2$

- To estimate the mean of a population, we could us the Sample mean ($\bar{X}$).
- Is the sample mean a good estimate?

Point Estimation

- Point Estimation- using a sample Statistic to estimate a population parameter
- Usually does a decent job (especially for larger sample sizes), but not perfect
- We can do better if we supplement that point estimate with what we know about the statistic’s variability

Confidence Intervals

- Remember, the goal here is to estimate the population mean, μ using the sample mean, $\bar{x}$

- We will use the results of the Central Limit Theorem to help us understand our one sample proportion.

- A confidence interval supplements an estimate of a parameter with an indication of its variability by using:

Point Estimate $\pm$ Margin of Error

- Confidence intervals supplement our point estimate with a "Margin of Error"

- They provide us with:
  - A range of plausible values for a population parameter.
  - A confidence level, which expresses our level of confidence that the interval contains the population parameter.

Example

- Estimate the average height of adult males in Virginia.

- We will take a sample of size 36.

- Calculate sample mean.

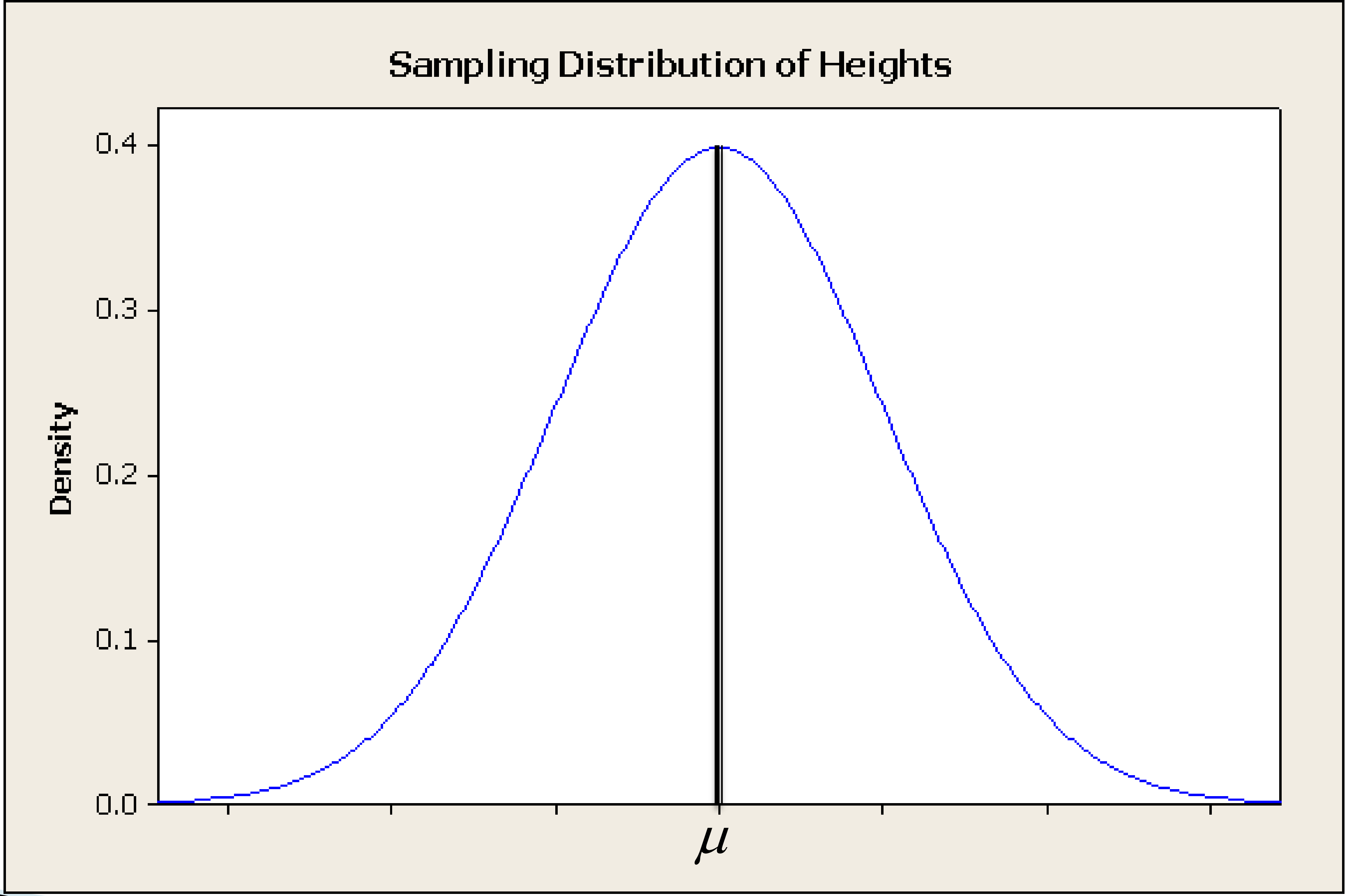
Assumptions

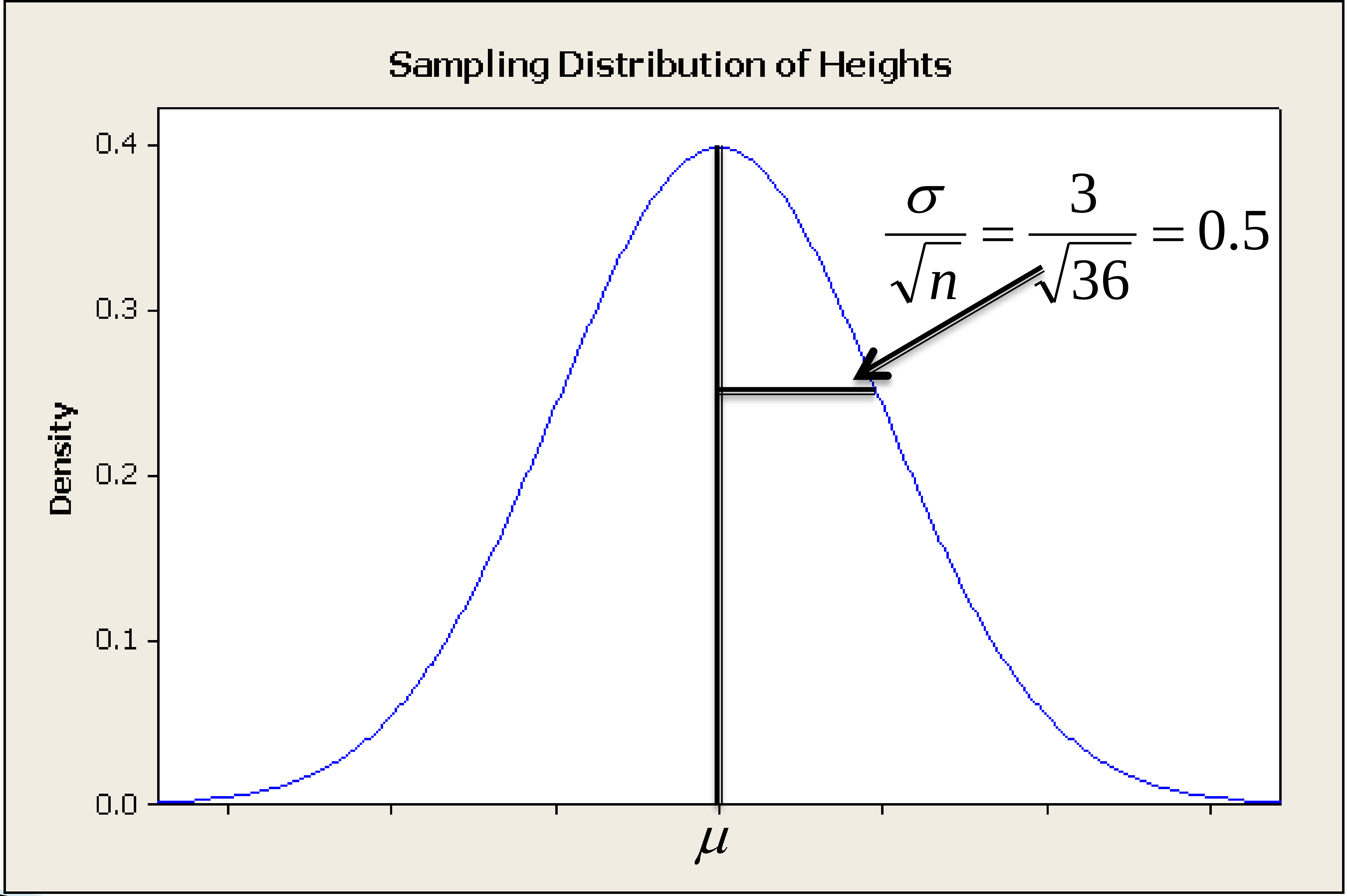
- Assume $\sigma = 3$
- Assume Central Limit Theorem properties hold…
  - The shape of the distribution of heights of one gender is approximately normal.
  - If not, our sample size of n = 36 is large enough.

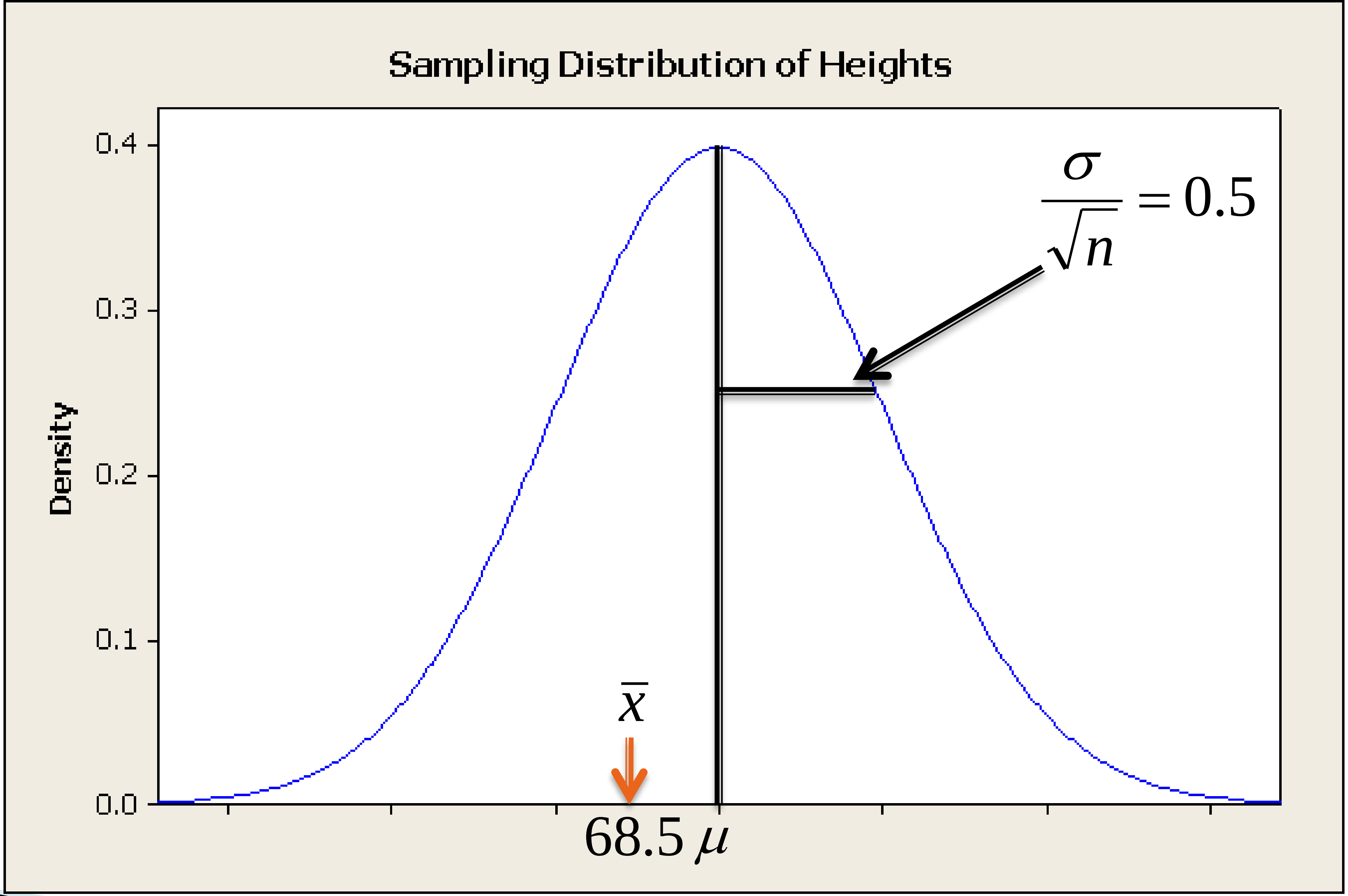
Sample of Heights (in inches)

60.3, 62.1, 65.4, 65.6, 65.7, 65.7,
65.9, 66.1, 66.1, 66.2, 66.4, 66.5,
67.5, 67.5, 67.7, 67.8, 67.9, 68.3,
68.3, 68.3, 68.5, 69.0, 69.2, 69.4,
69.5, 69.8, 70.1, 70.6, 70.8, 71.0,
71.5, 71.7, 72.5, 72.6, 74.4, 79.2

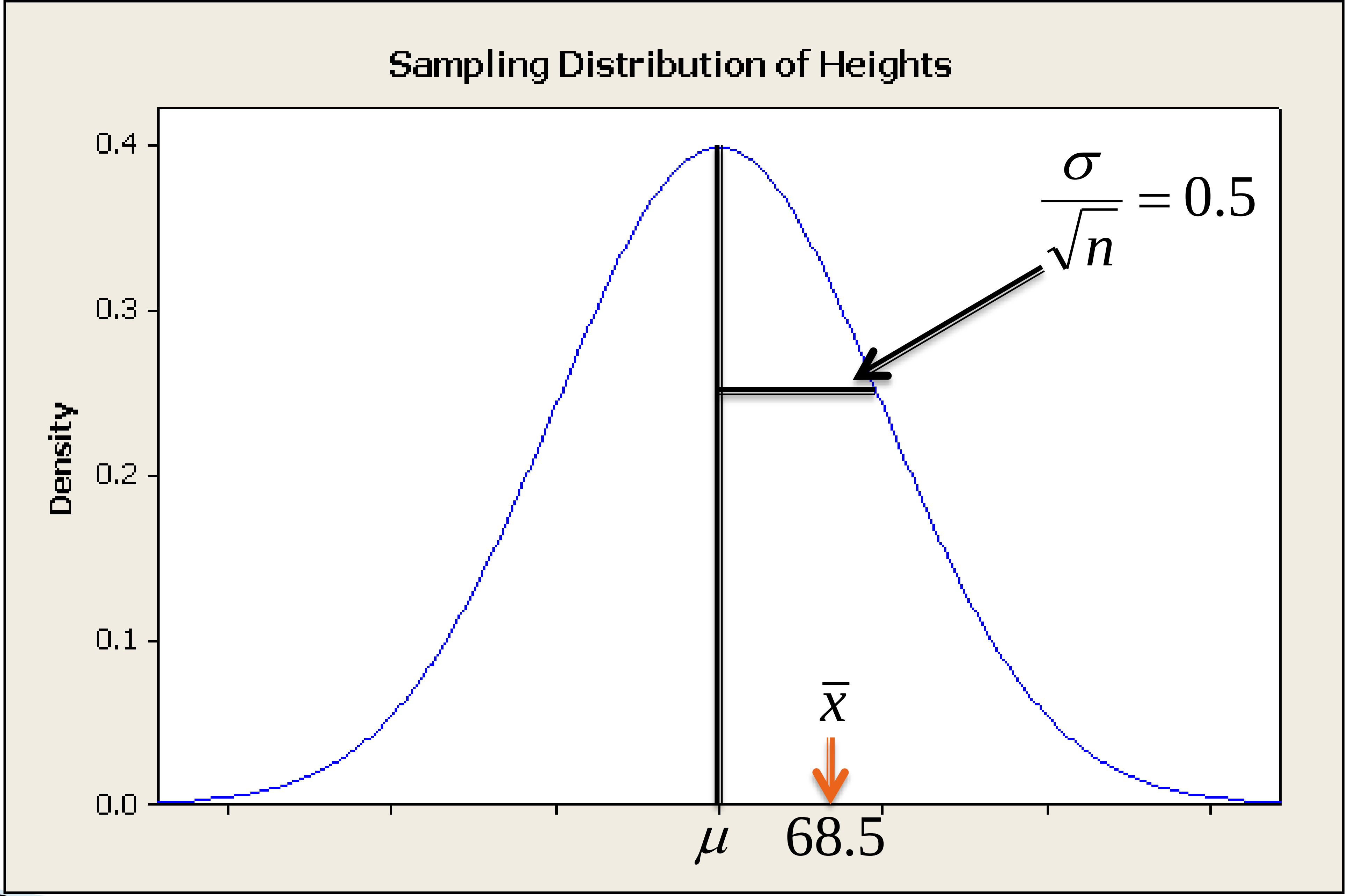
- Sample mean, $\bar{x} = 68.5$
- Population mean, μ is unknown

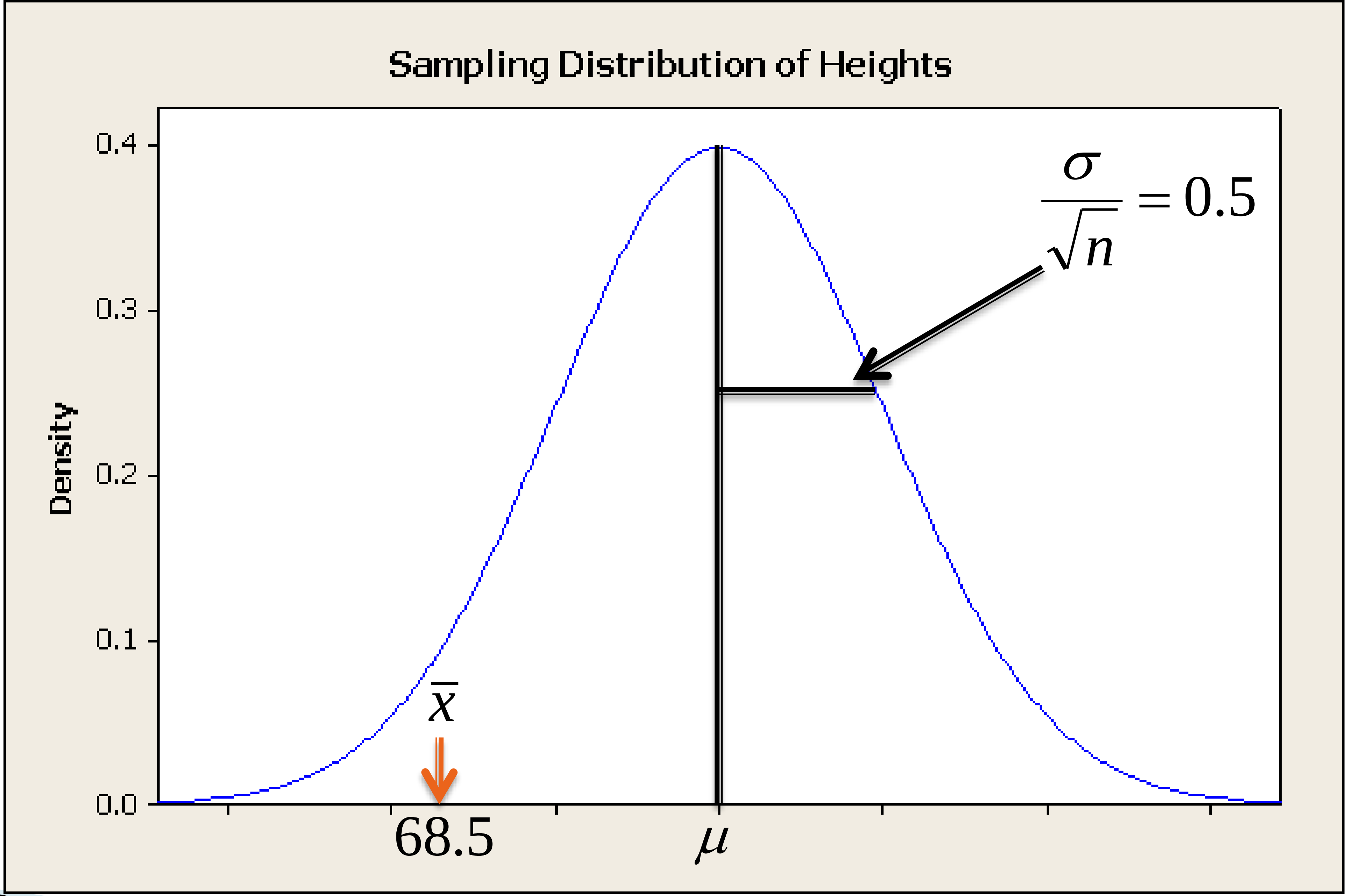

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
μ

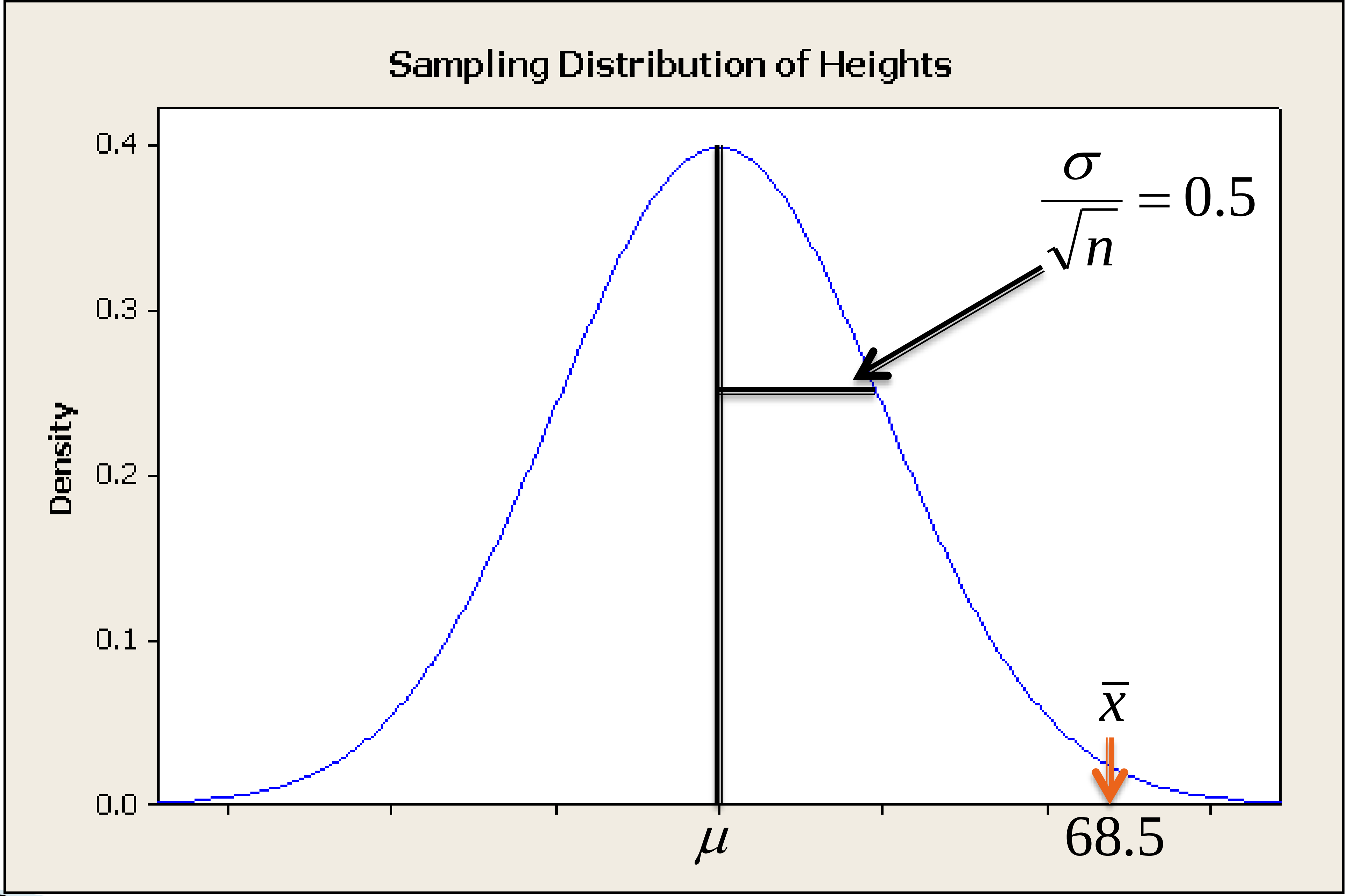

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
$\frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{36}} = 0.5$
μ



Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
$\frac{\sigma}{\sqrt{n}} = 0.5$
$\bar{x}$
68.5 μ

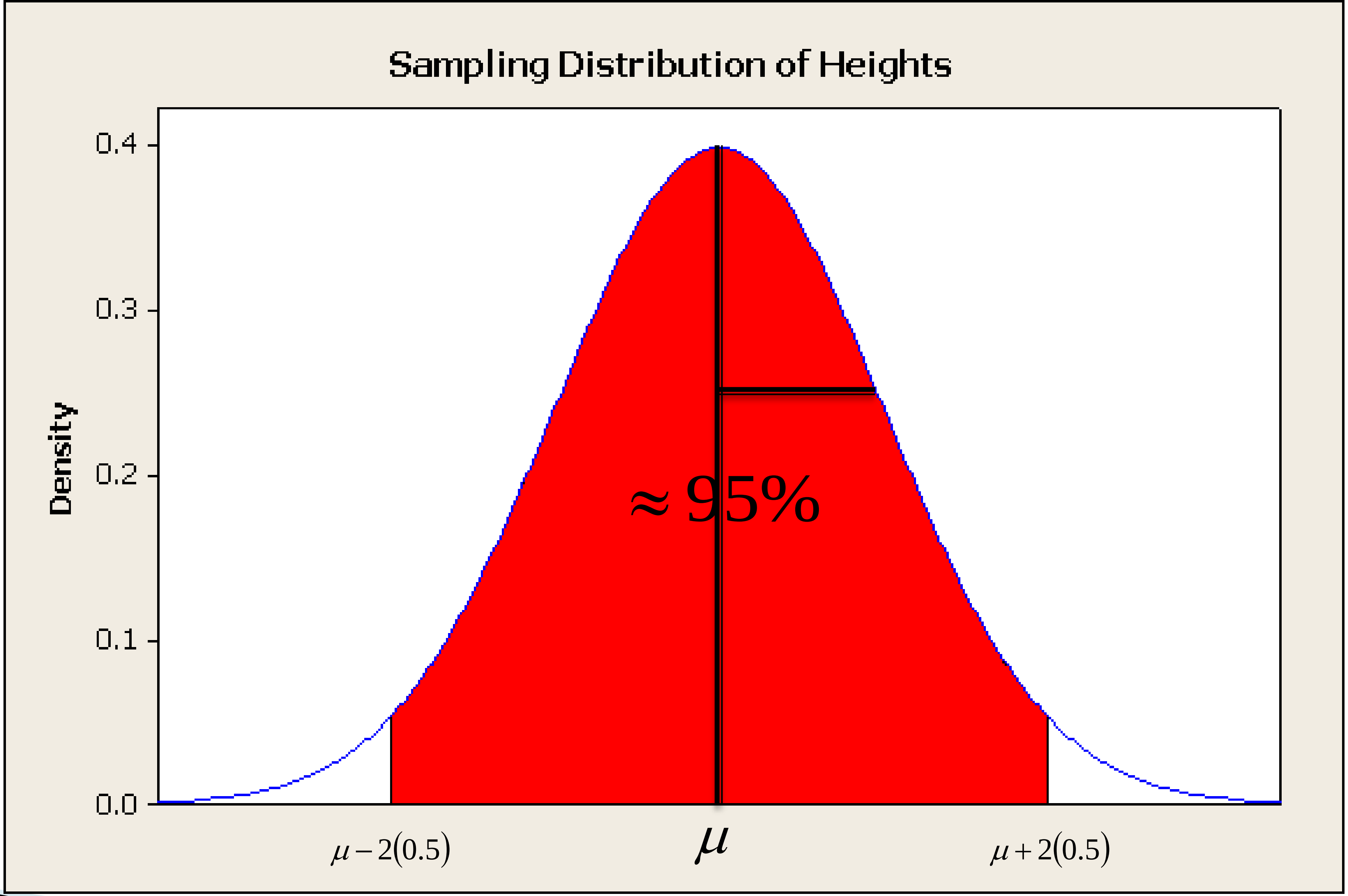

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
$\frac{\sigma}{\sqrt{n}} = 0.5$
$\bar{x}$
μ
68.5


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
$\frac{\sigma}{\sqrt{n}} = 0.5$
$\overline{x}$
68.5
μ


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
$\frac{\sigma}{\sqrt{n}} = 0.5$
$\bar{x}$
μ
68.5

Property of the Normal Distribution

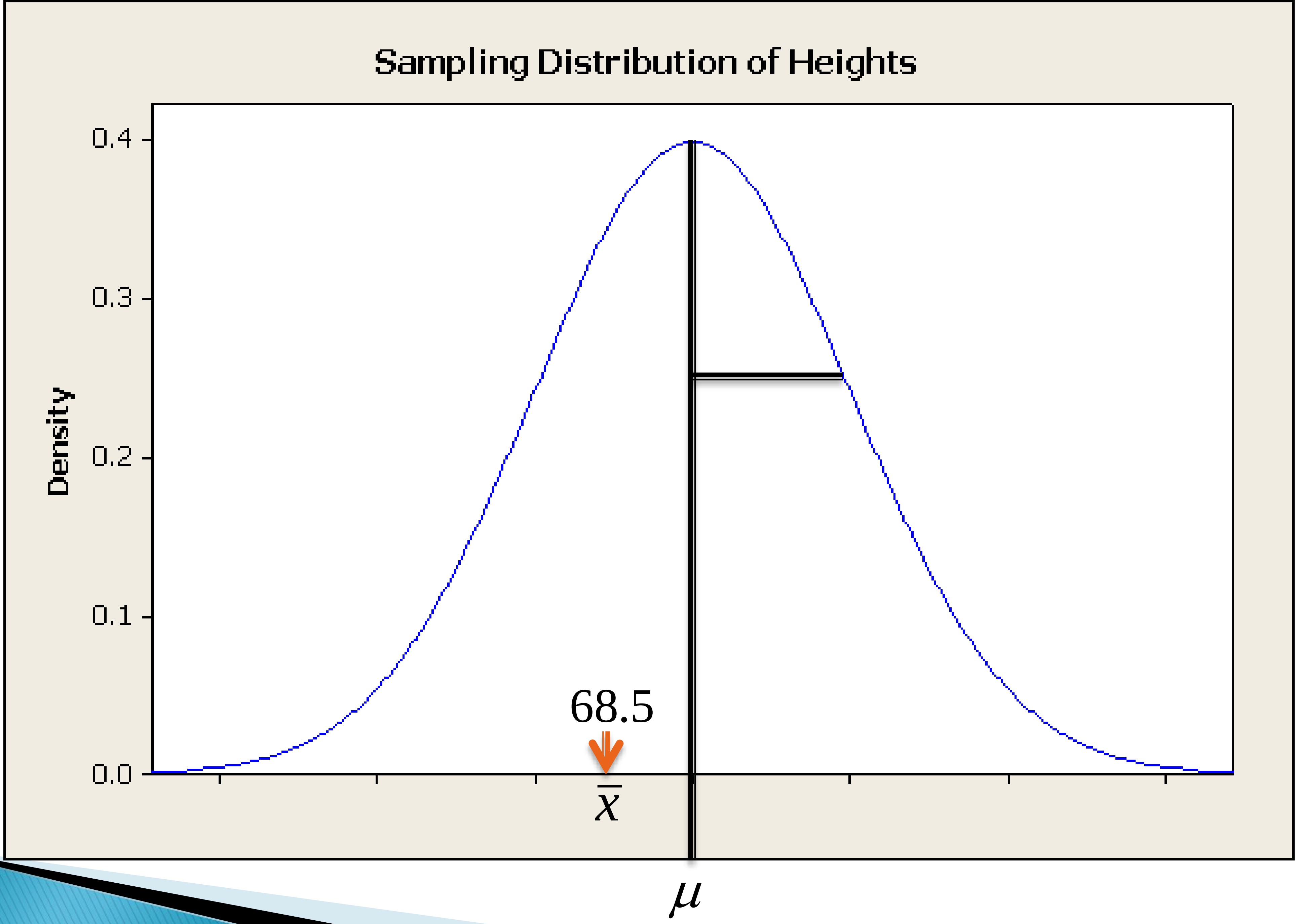
- Think about the "Empirical rule" in this situation
- If we go two units of standard deviation away from the population mean, we will cover approximately 95% of the sample means
- See image…

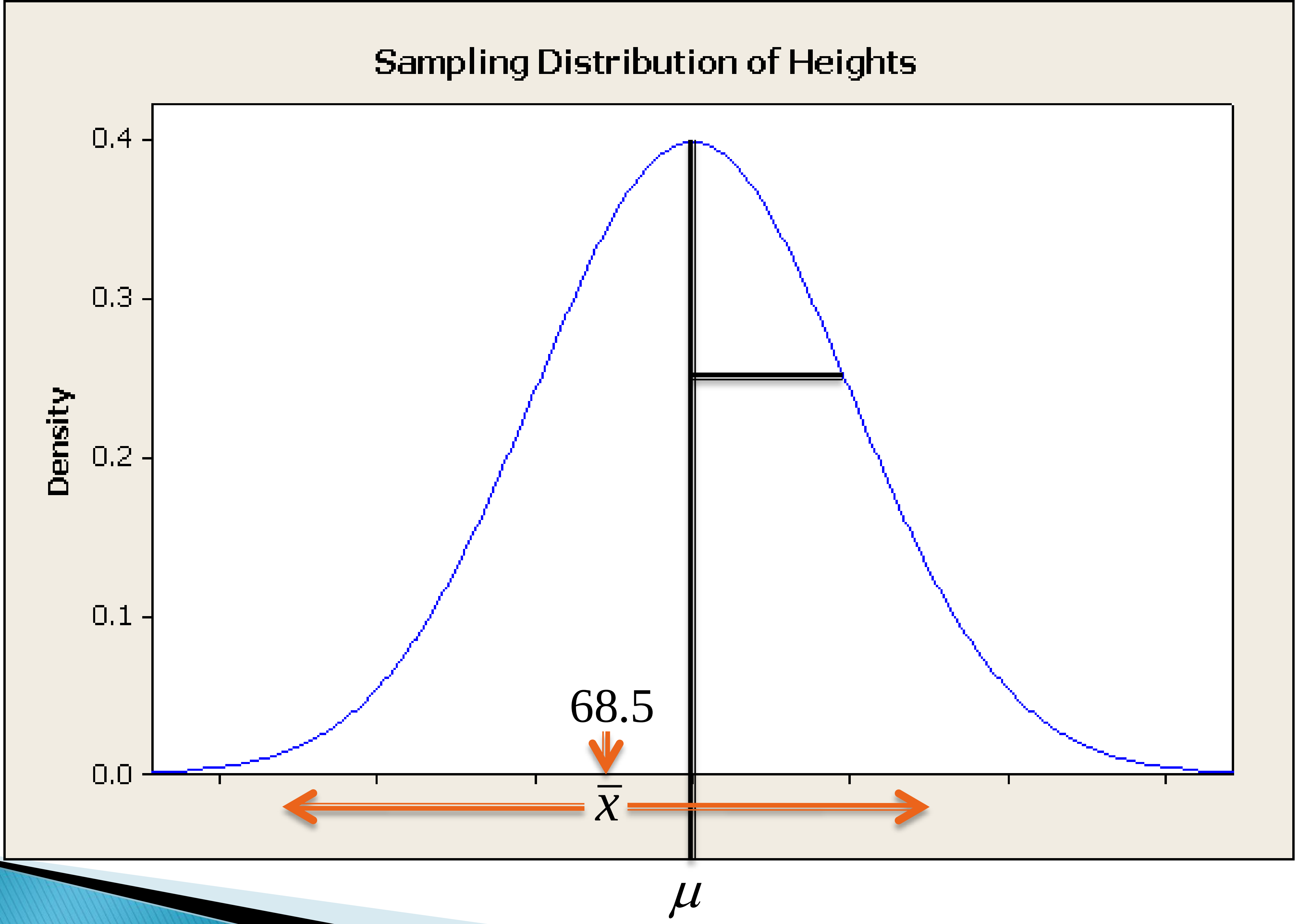

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
≈ 95%
μ − 2(0.5)
μ
μ + 2(0.5)

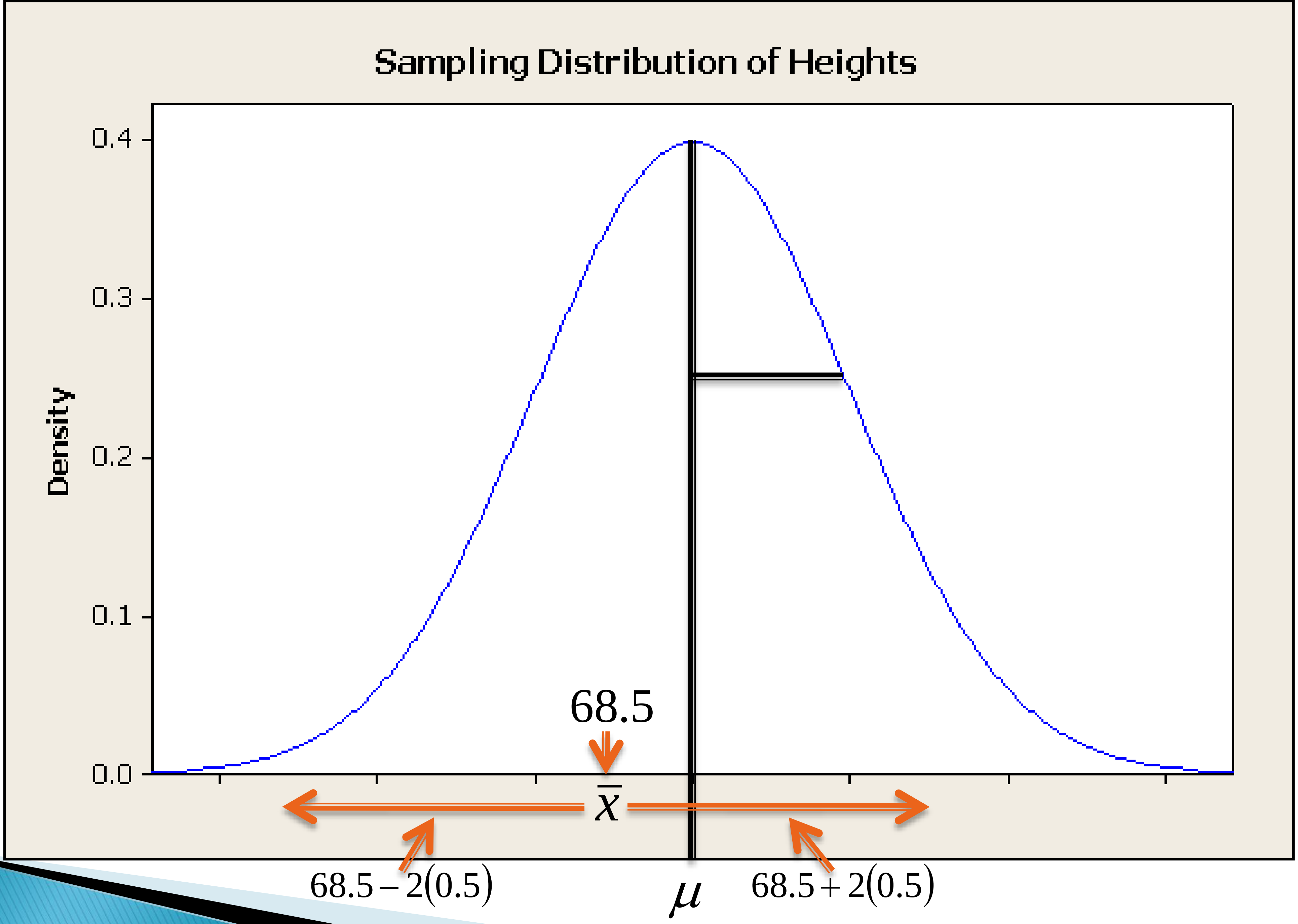
Information

- There is roughly a 95% chance that μ is no more than 2 standard deviations from $\bar{x}$

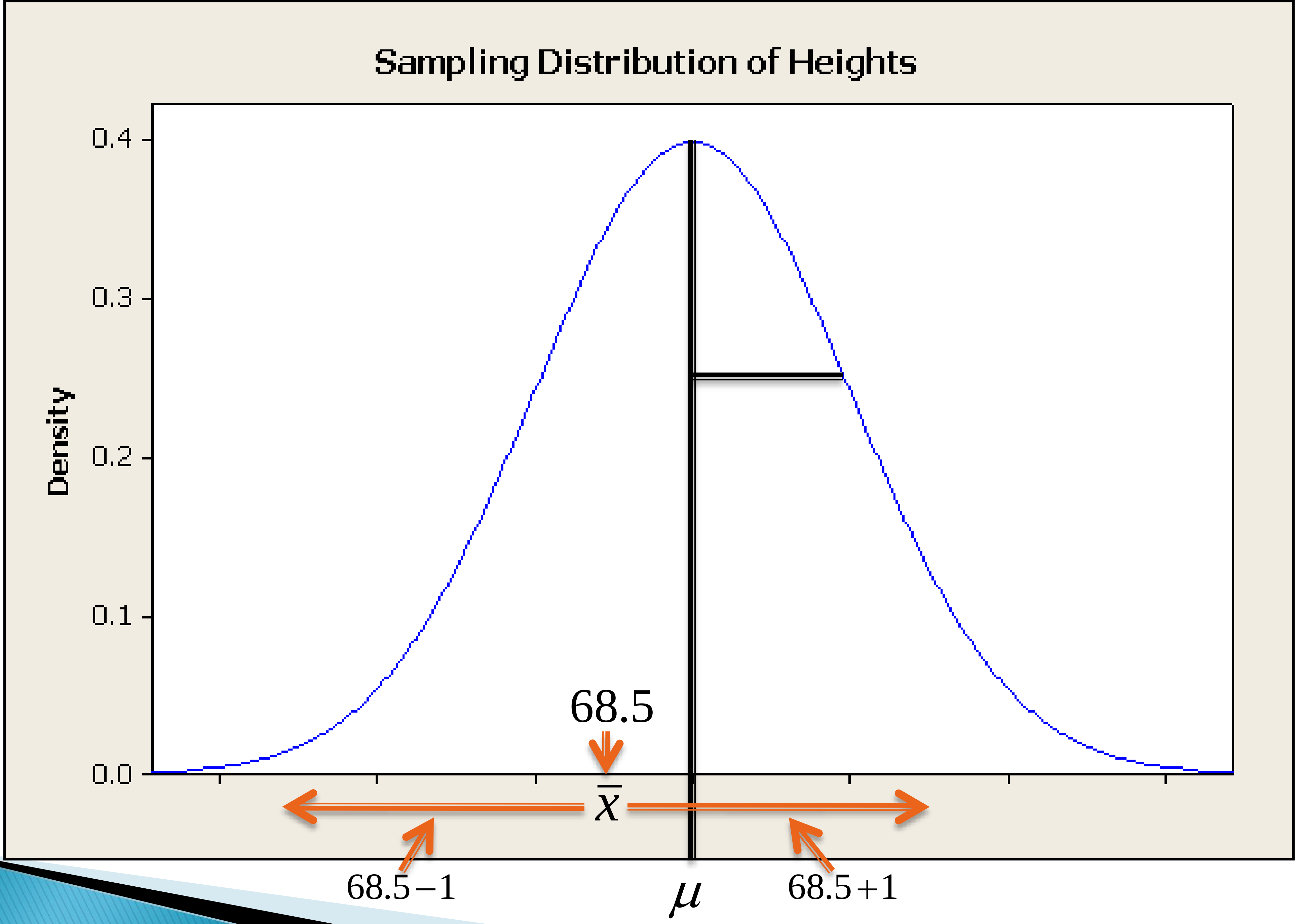
- Now, if we reach out 2 SDs away from our $\bar{x}$ of 68.5 on both sides we are roughly 95% sure that the unknown quantity, μ will be captured in this interval.

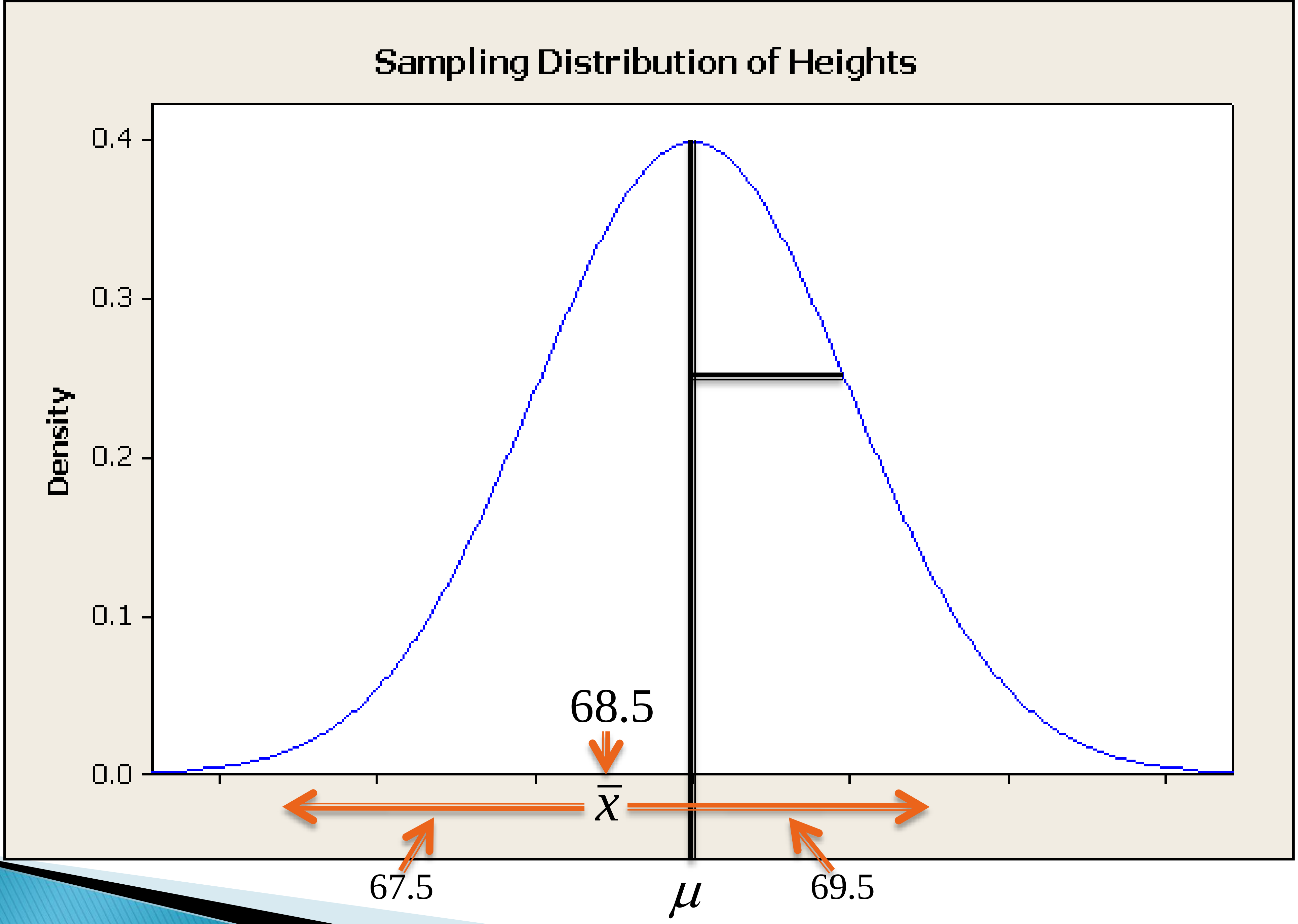

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
μ


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
μ

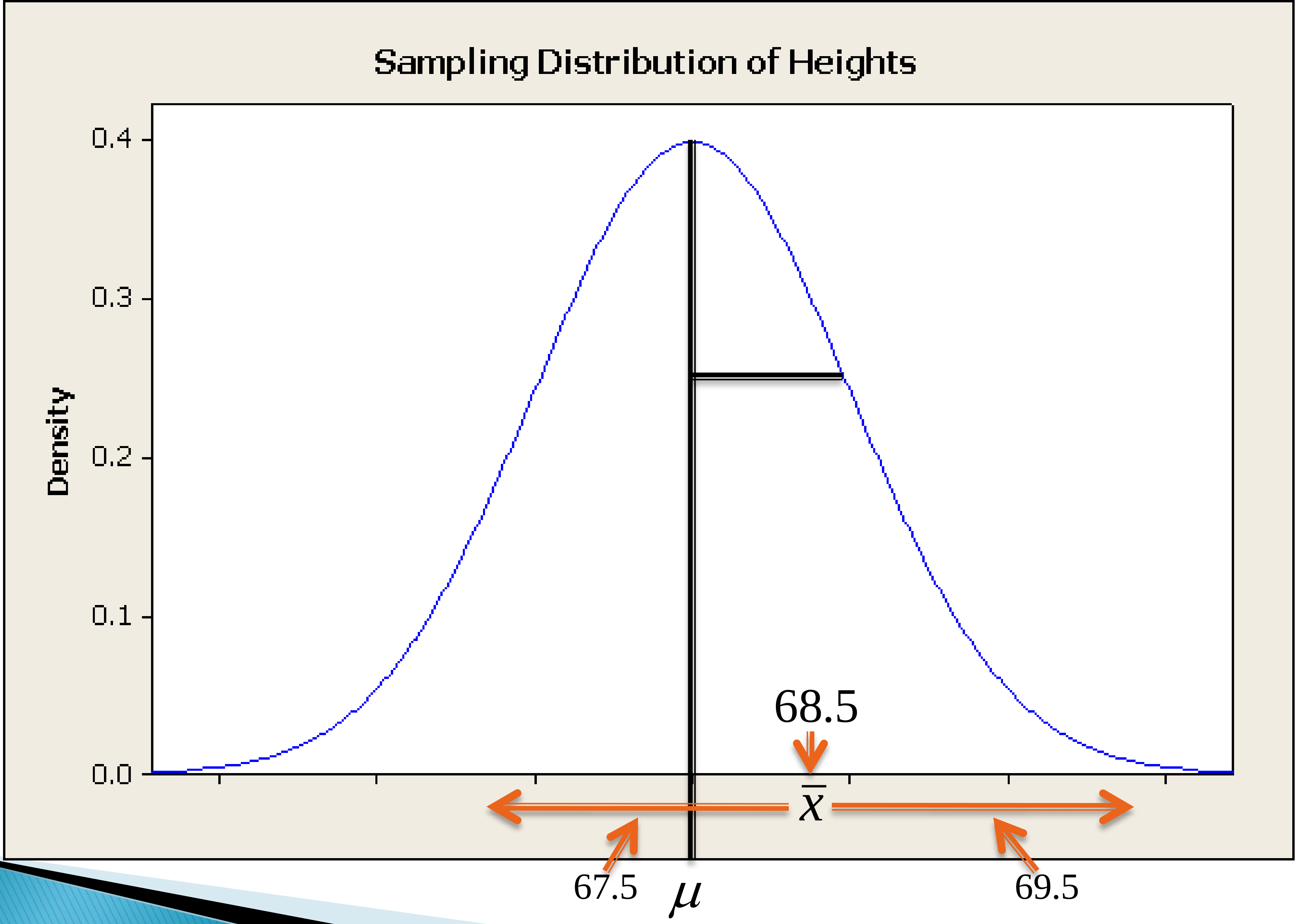


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
$68.5-2(0.5)$
μ
$68.5+2(0.5)$

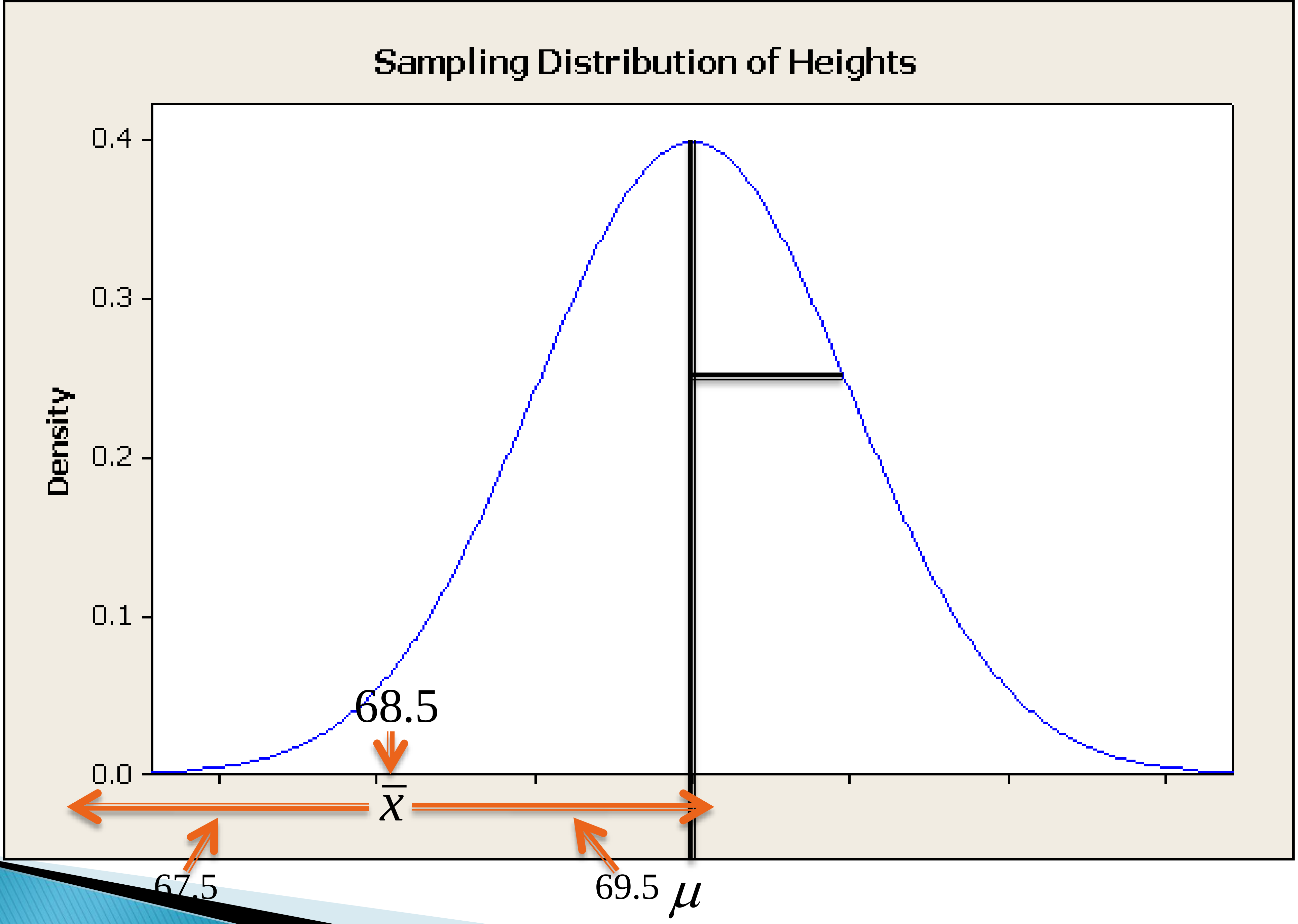

Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
68.5−1
μ
68.5+1

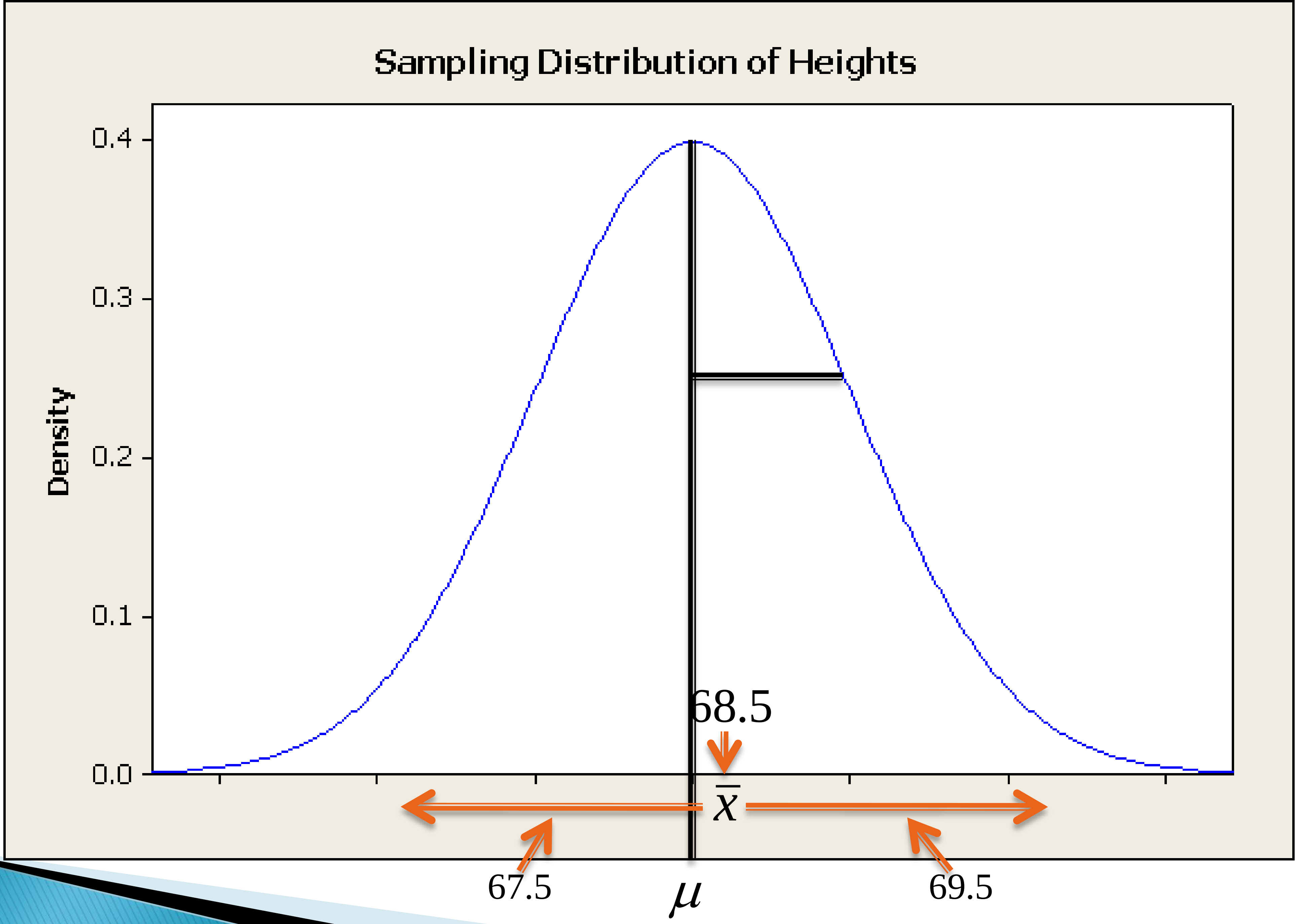


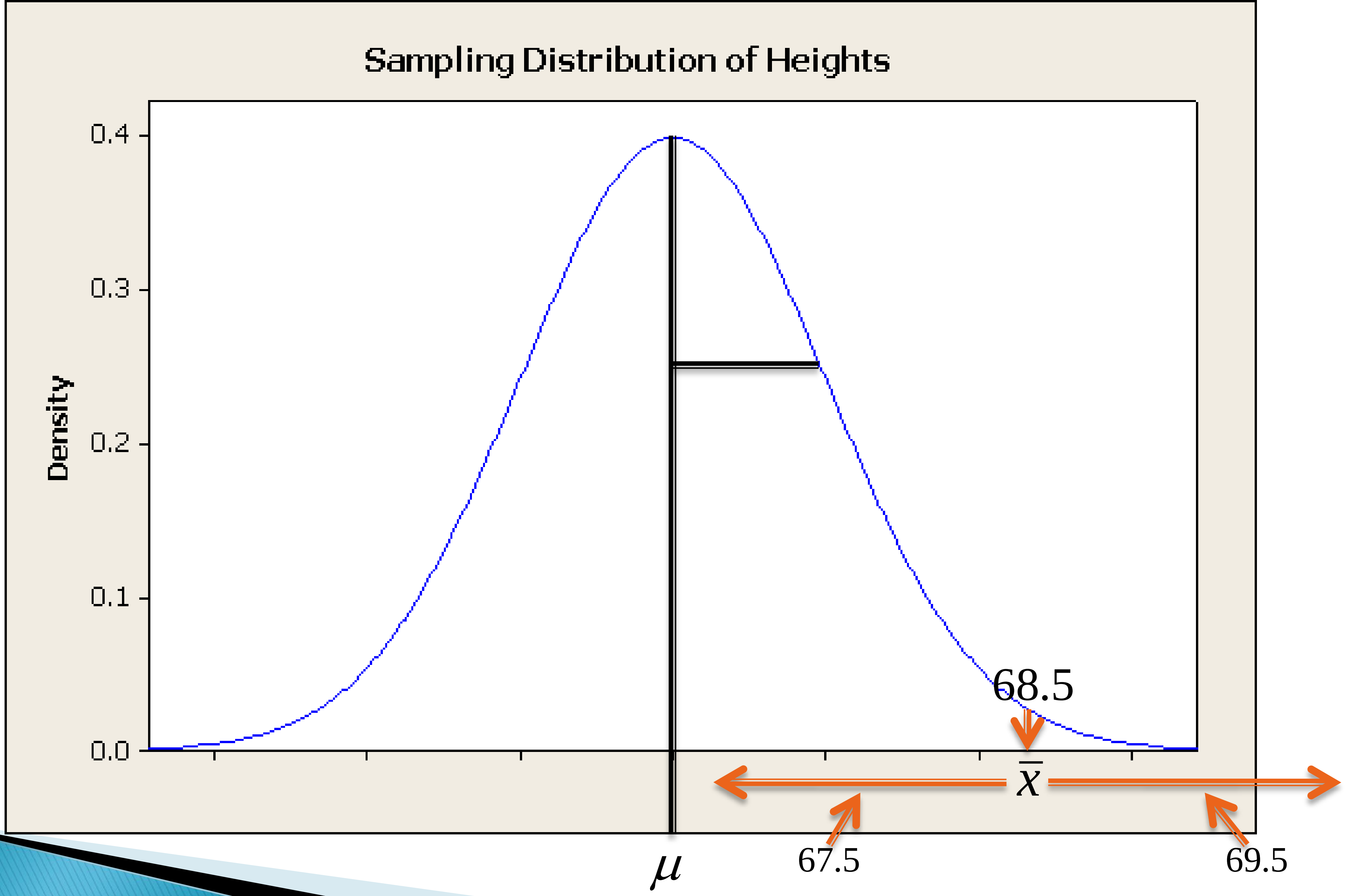
Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
67.5
μ
69.5



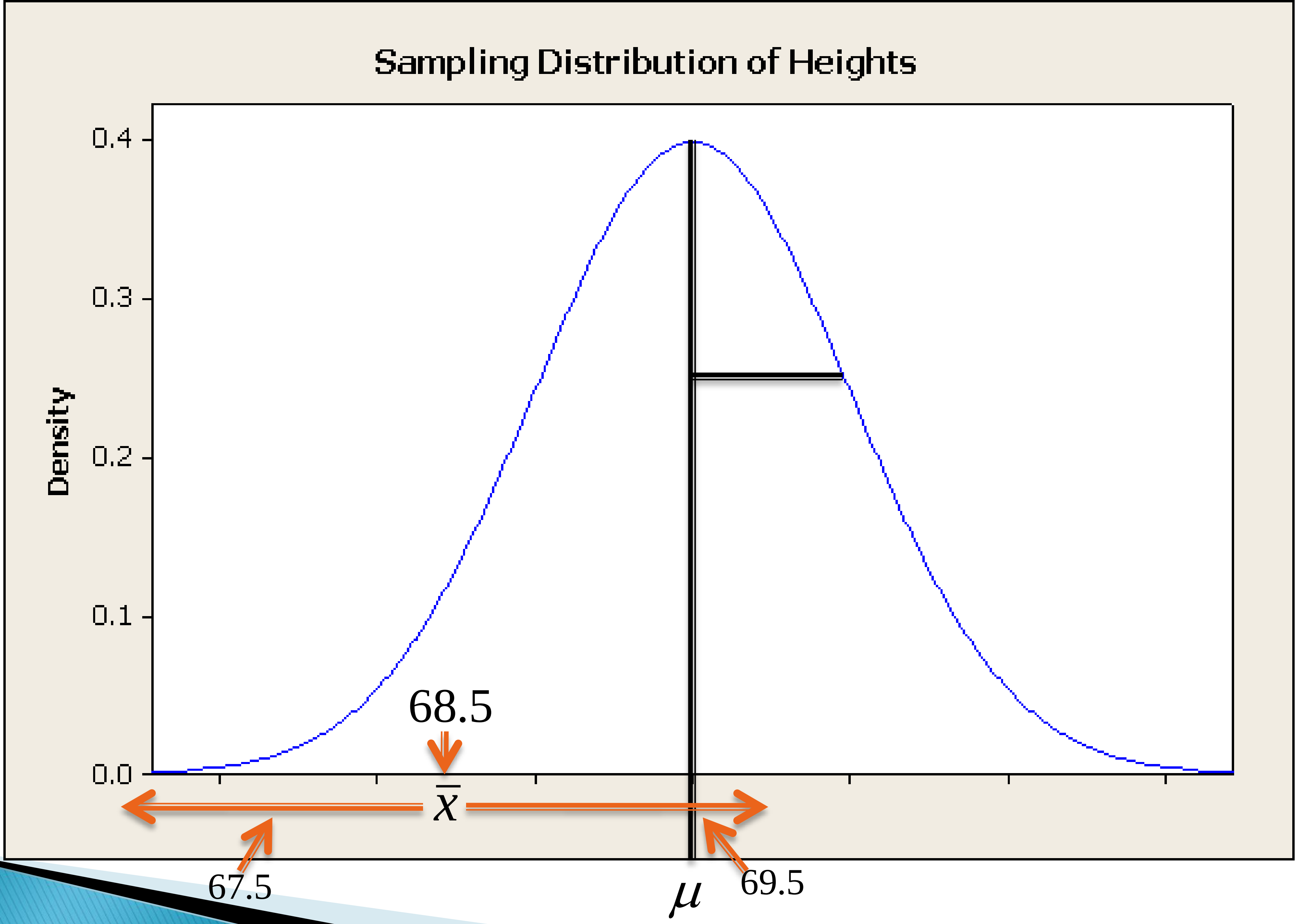
Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
67.5
μ
69.5


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
67.5
69.5
μ


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
67.5
μ
69.5



Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
μ
67.5
69.5


Sampling Distribution of Heights
Density
0.4
0.3
0.2
0.1
0.0
68.5
$\bar{x}$
67.5
μ
69.5

95% Confidence Interval

- (67.5, 69.5)
- We are roughly 95% sure that this interval captures the unknown population mean height.
- Supplements our estimate of μ with an indication of $\bar{x}$'s variability.

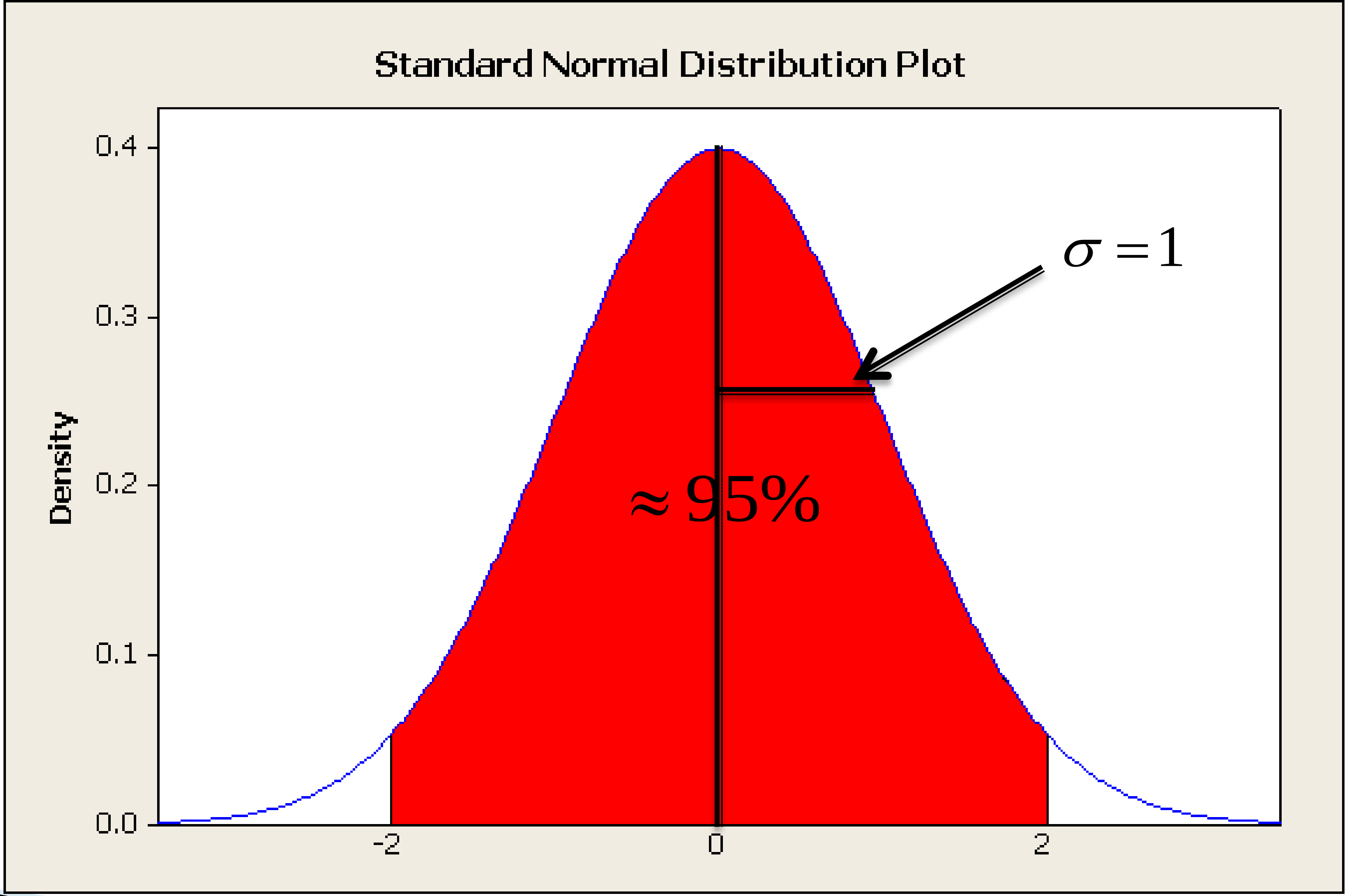
95% Confidence Interval

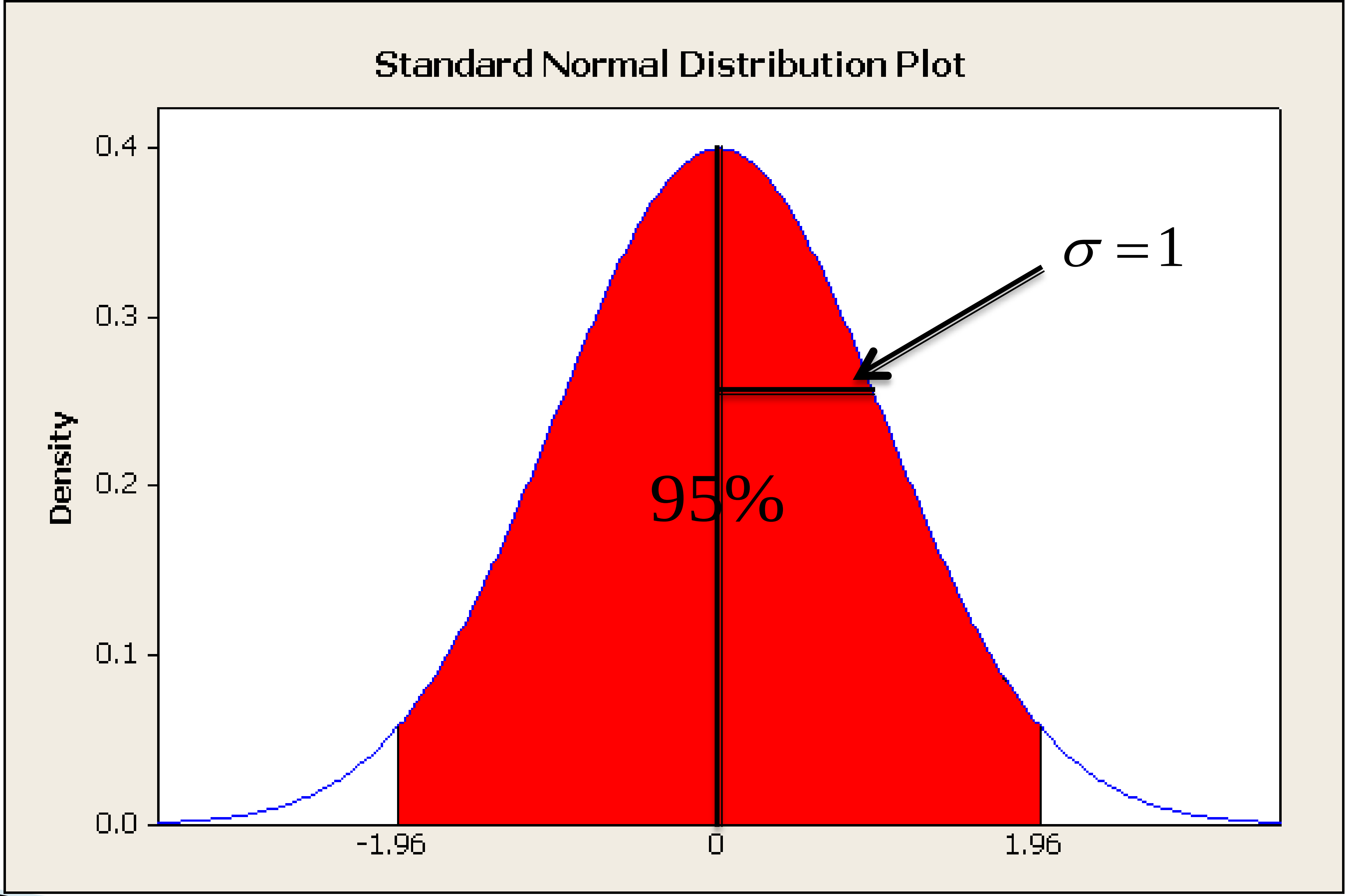
- (67.5, 69.5)

- Formula:

$$\bar{x} \pm 2\left(\frac{\sigma}{\sqrt{n}}\right)$$

- The 2 in the formula is an approximate value and will change depending on our level of confidence.


Standard Normal Distribution Plot
Density
0.4
0.3
0.2
0.1
0.0
-2
0
2
$\sigma = 1$
$\approx 95\%$

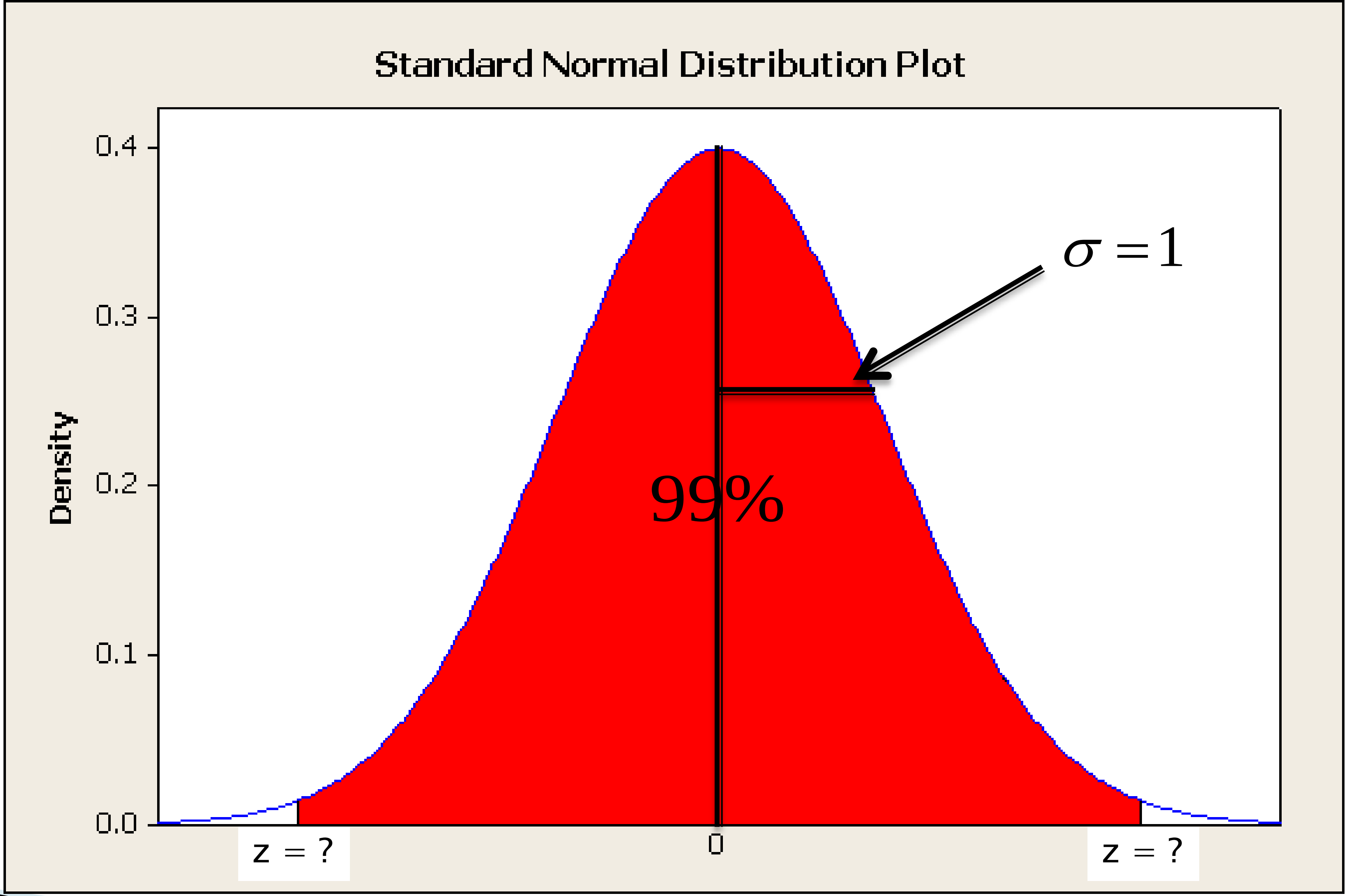

Standard Normal Distribution Plot
Density
0.4
0.3
0.2
0.1
0.0
-1.96
0
1.96
$\sigma = 1$
95%

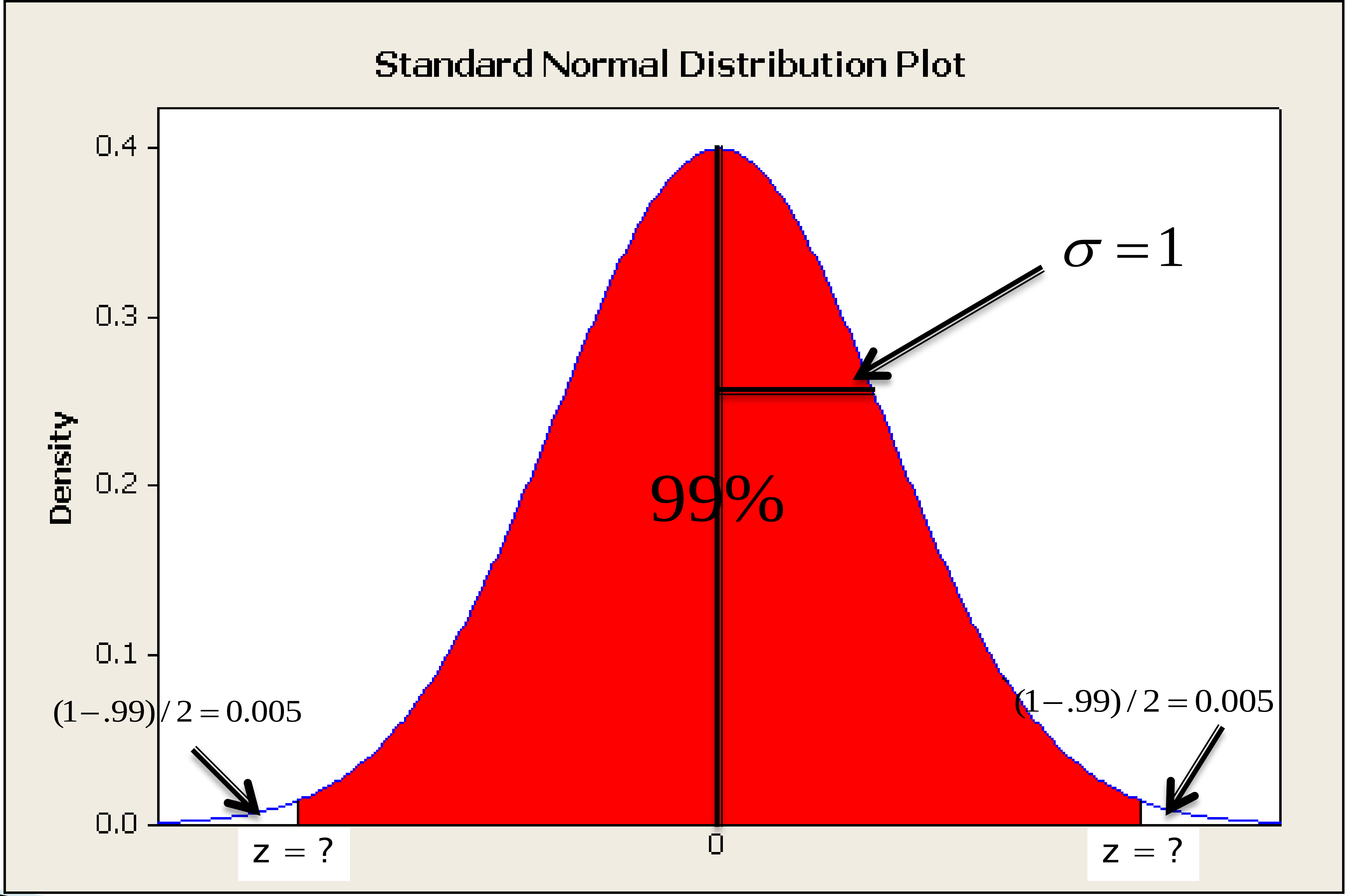
95% Confidence Interval

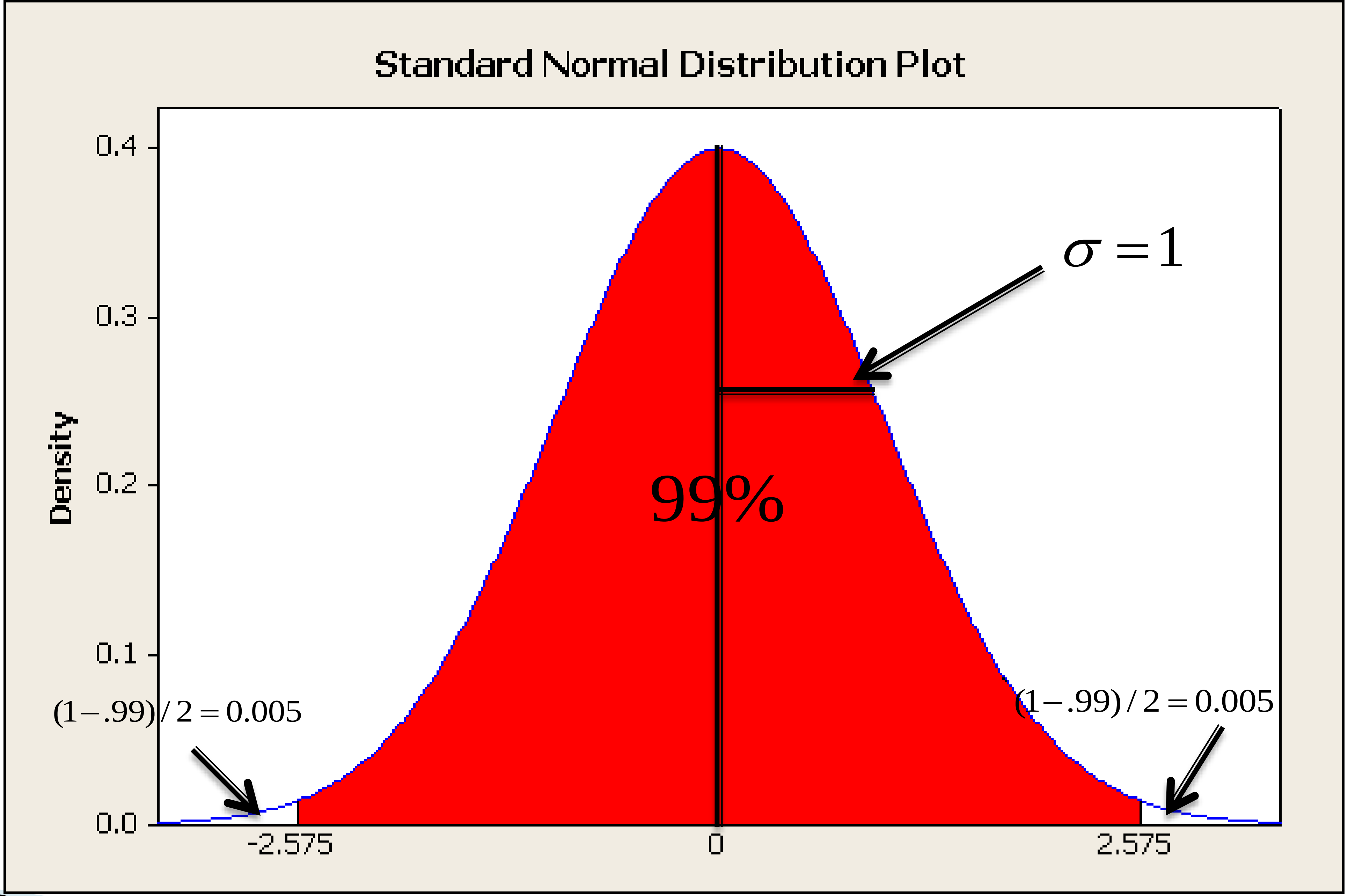
- Formula:

$$\bar{x} \pm 1.96\left(\frac{\sigma}{\sqrt{n}}\right)$$

Critical Value


Standard Normal Distribution Plot
Density
0.4
0.3
0.2
0.1
0.0
$\sigma = 1$
99%
z = ?
0
z = ?


Standard Normal Distribution Plot
Density
0.4
0.3
0.2
0.1
0.0
$\sigma = 1$
99%
$(1-.99)/2 = 0.005$
$(1-.99)/2 = 0.005$
z = ?
0
z = ?


Standard Normal Distribution Plot
Density
0.4
0.3
0.2
0.1
0.0
-2.575
0
2.575
$\sigma = 1$
99%
$(1-.99)/2 = 0.005$
$(1-.99)/2 = 0.005$

Critical Values

- Think about the area in the middle of the distribution as we did above and then use it to find the critical value.

- If area CL is found in the middle of our standard normal distribution, then the area $\alpha = 1 - CL$ represents the total area in both tails.

- Thus, the area in one tail would be $\frac{\alpha}{2}$

- Usually denoted $z_{\frac{\alpha}{2}}$ where $\alpha = 1\text{–}CL$ or z*

Finding Critical Values

- Commonly we use 95%, 99%, and 90%, but sometimes you may have to find it on your own using the table or technology

- I recommend drawing a picture, using the table, then checking yourself with a Normal Calculator

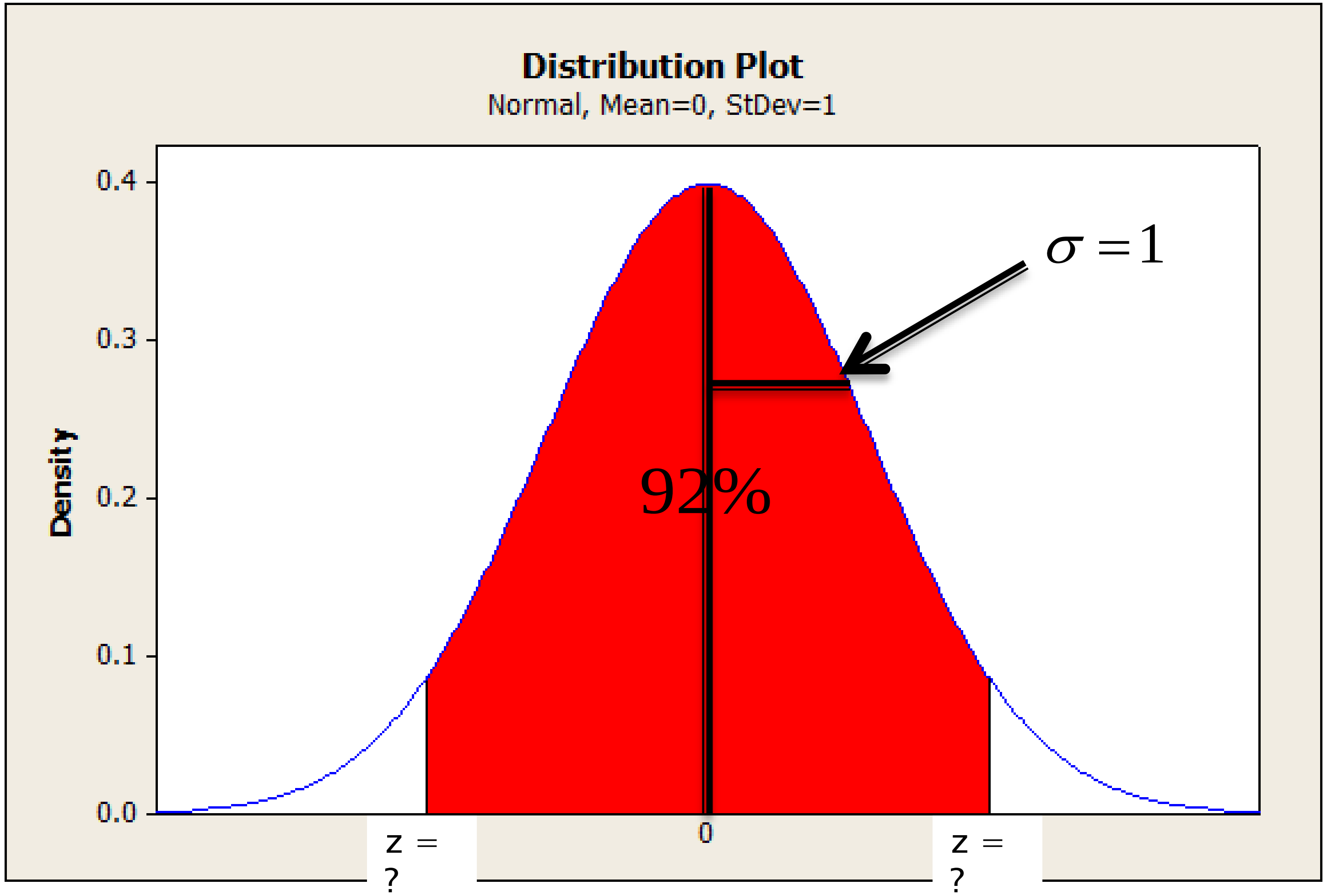
Procedure for finding Critical Value

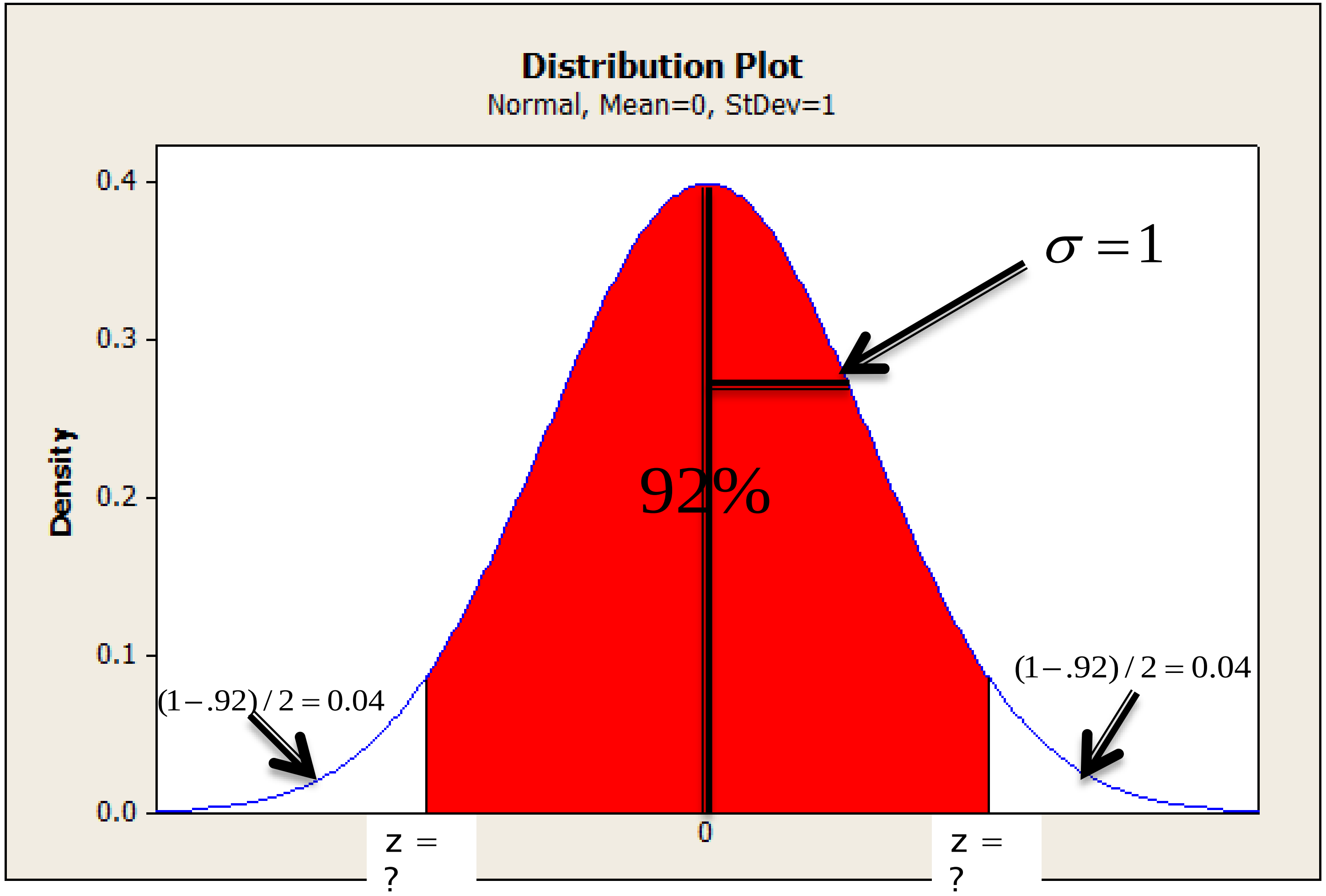
- Ex: Find the critical value for a 92% confidence interval.

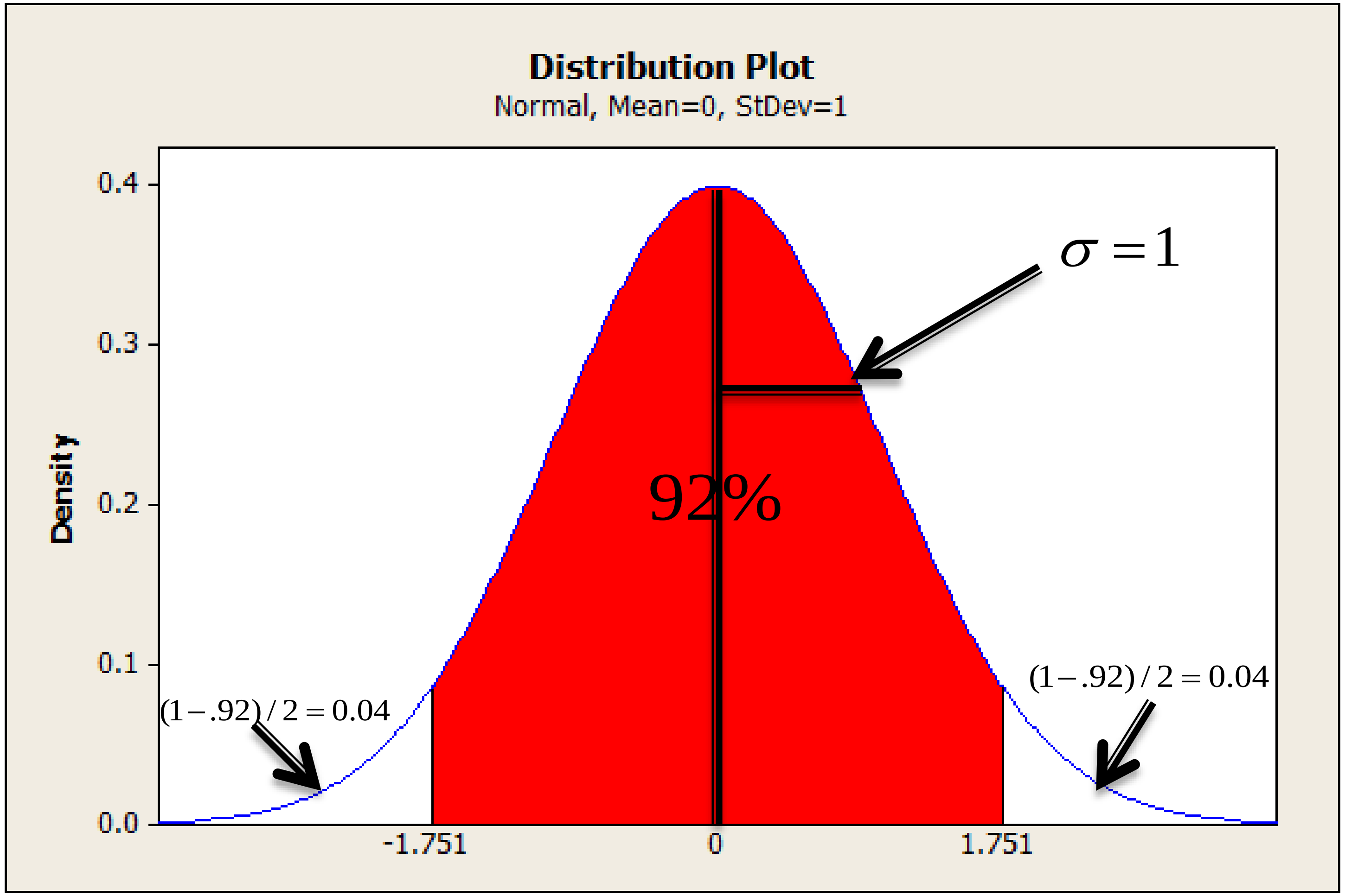
- Draw picture (see board…)

- CL = 0.92 => 1 – CL = α = 0.08 => $\frac{\alpha}{2} = \frac{0.08}{2} = 0.04$

- Look up 0.04 in the Z table to find the critical value. Verify it using StatCrunch.


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
$\sigma = 1$
92%
z = ?
0
z = ?


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
$\sigma = 1$
92%
$(1-.92)/2 = 0.04$
$(1-.92)/2 = 0.04$
0
z = ?
z = ?


Distribution Plot
Normal, Mean=0, StDev=1
Density
0.4
0.3
0.2
0.1
0.0
-1.751
0
1.751
92%
$\sigma = 1$
$(1-.92)/2 = 0.04$
$(1-.92)/2 = 0.04$

Rationale for a Confidence Interval

- To develop a confidence interval estimate, let *L* and *U* (lower & upper) be random variables whose sample values (*l* and *u*) are the lower & upper confidence limits.
  - Remember, μ is fixed.

$$P\{L \le \mu \le U\} = 1 - \alpha \qquad (8\text{-}2)$$

The confidence interval is $l \le \mu \le u$ (8-3)

The confidence coefficient is $1 - \alpha$

with $0 \le \alpha \le 1$

Rationale for a Confidence Interval

Let $X_1, X_2, \ldots X_n$ denote a random sample from a normal distribution with unknown mean μ and known variance σ^2. Then $\bar{X}$ is normally distributed with mean μ and variance σ^2/n.

$$\bar{X} \text{ is standardized as } Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \qquad (8\text{-}1)$$

where Z is a standard normal random variable.

Rationale for a Confidence Interval

$$P\left\{-z_{\alpha/2} \le Z \le z_{\alpha/2}\right\} =$$

$$P\left\{-z_{\alpha/2} \le \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \le z_{\alpha/2}\right\} =$$

$$P\left\{\bar{X} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{X} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right\} = 1-\alpha \qquad (8\text{-}4)$$

$$\text{where } \bar{X} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = l \text{ and } u$$

Building a CI

Put simply In words:

estimate $\pm$ (critical value) x (standard error)

Using:

$$\bar{X} \sim N(\mu, \frac{\sigma}{\sqrt{n}})$$

Then:

$$\bar{X} \pm Z^{*}(\frac{\sigma}{\sqrt{n}})$$

CI for a Population Mean (with σ Known) Requirements

- 1. The sample is a simple random sample.
- 2. The value of the population standard deviation is known.
- 3. Either or both of these conditions is satisfied: The population is normally distributed or $n > 30$

Confidence Interval Example

- The Charpy V-notch technique, used for impact testing of metallic materials, measures impact energy contributing to a ductile-to-brittle transition with the decreasing temperature of materials. Assume the population we are sampling from is Normal and σ=3
- Calculate a 95% confidence interval to estimate the mean impact energy (joules)

$$\bar{x} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$64.46\text{-}1.96\frac{1}{\sqrt{10}} \le \mu \le 64.46\text{+}1.96\frac{1}{\sqrt{10}}$$

$$63.84 \le \mu \le 65.08$$

Data in joules	
64.1	$= x_i$
64.7	
64.5	
64.6	
64.5	
64.3	
64.6	
64.8	
64.2	
64.3	n =10
64.46	= x-bar
0.23	= s
1.00	= σ
0.05	= α
1.96	= z(α/2)

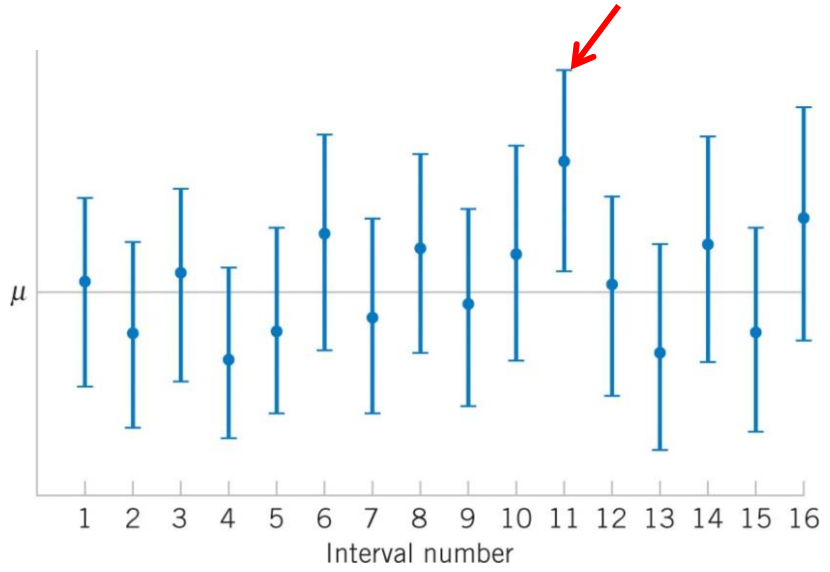
Interpreting a Confidence Interval

- It is improper to say: “The probability is 95% that μ is in the {63.84, 65.08} interval.” μ is a fixed number, not a random variable.
- The bounds {U, L} are the random variables.

- It is proper to say:
  - “There is about a 95% chance that the interval calculated is one that will include μ.”

    OR

  - “If many intervals are created from many samples, about 95% of those intervals will include μ” or



Confidence Level & Precision of Estimation

- The level of confidence $(1-\alpha)$ of 95% is an arbitrary choice creating an interval length of 1.24 (63.84 to 65.08).

- If the level of confidence is increased to 99% ($\alpha=0.01$), then the interval length increases to:

$$\text{Interval length} = 2 \cdot z_{0.01/2} \frac{\sigma}{\sqrt{n}} = 2(2.576)\frac{1}{\sqrt{10}} = 1.63$$

- Therefore, the level of confidence varies directly with the interval length

Sample Size with the *Z* Confidence Interval for μ

- What does the behavior of the MoE look like?

- For a desired margin of error (*ME*), we can calculate the sample size.

$$ME = z^* \left(\frac{\sigma}{\sqrt{n}} \right)$$

- Solving for n:

$$n = \left(\frac{z^* \sigma}{ME} \right)^2$$

- Always round *n* to the next whole number.

Factor Relationships

- There are 4 factors in the sample size calculation.
- Fixing the value of any 2 factors makes the other 2 relate directly or inversely.

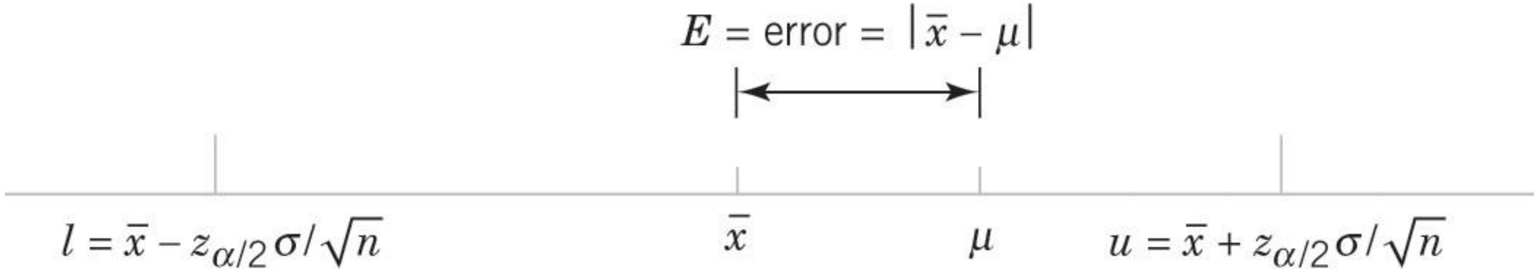
Relationships of:			
Interval Length (2*E*)	Sample Size (*n*)	Confidence $(1 - \alpha)$	Std Dev (σ)
$\searrow$	$\nearrow$	fixed	fixed
$\nearrow$	fixed	$\nearrow$	fixed
fixed	$\nearrow$	$\nearrow$	fixed

Choice of Sample Size

- The precision (1−α) and the length of the confidence interval are related to the sample size.
- If *x-bar* is used as an estimate of μ, we can be 100(1− α)% confident that the error |*x-bar* − μ| will not exceed a specified amount *E* when the sample size is:

$$n = \left(\frac{z_{\alpha/2}\sigma}{E} \right)^2 \qquad (8\text{-}6)$$

- If n is not an integer, round up to maintain the level of confidence.



Changing Interval Size Ex.

- Using the data from the last example

- Now, we want to determine how many specimens must be tested to ensure that the 95% *CI* on μ has a length of at most 1.0*J*(joules), or $E=0.5$.

$$n=\left(\frac{z_{\alpha/2}\sigma}{E}\right)^2=\left[\frac{(1.96)1}{0.5}\right]^2=15.37$$

- Always rounding up to ensure the level of confidence, $n = 16$

Statistical Inference Methods

- Statistical Inference: Drawing conclusions about a population from sample data.

- Methods
  - Point Estimation – Using a sample statistic to estimate a parameter
  - Confidence Intervals – supplements an estimate of a parameter with an indication of its variability
  - Hypothesis Tests– assesses evidence for a claim about a parameter by comparing it with observed data

- Because a different sample might lead to different conclusions, we cannot be certain that our conclusions are 100% correct.
  - Statistical inference uses the language of probability to say how reliable our conclusions are.

Different Methods

- Even though confidence intervals and hypothesis testing are both inference methods, and share some terminology and notation, they serve different purposes

- A confidence interval answers the question "what is the value of the parameter?"

- A hypothesis test is used to judge a claim about the parameter.

Hypothesis Testing Rationale

- The amount of sleep most people should get is about 8 hours of sleep per night.

- You think students are not getting enough sleep and want to measure student's sleep patterns.

- How much sleep would your sample of students have to get, on average, to convince you (and others) your hypothesis is correct?

- The big questions are:
  - Where do you set the "cut off" value for your evidence?
  - Are the results you observed simply due to variation or the sample you took?

Hypothesis Testing

Purpose – to aid investigators in reaching a decision concerning a population by examining the data obtained from a sample

Goal – to assess the evidence provided by the data in favor of some claim about the population

Definitions

Hypothesis Test – Decision making process for evaluating claims about a population

1) State the appropriate hypotheses
2) State the appropriate test statistic
3) State the Critical Region
4) Conduct the experiment and calculate the test statistic
5) Draw your conclusion

Definitions

Hypothesis – an assumption made about the population from which the sample was taken.

Two Types

1) Null Hypothesis, H_0 (read H–naught)
2) Alternative Hypothesis, H_a (read H–A)

Null Hypothesis, H_0

- It is the "no change" or "no difference" statement about a population parameter.

- This is our starting point that we must assume is true in order to show otherwise.

- The hypothesis test is designed to assess the strength of the evidence against the null hypothesis.

- In our example:
  - In words: Student's are getting the recommended amount of sleep
  - H_0: $\mu = 8$

Alternative Hypothesis, H_a

- The research hypothesis; the statement about a population parameter we intend to demonstrate is true.
- Claims that the effect we are looking for does exist.
- In our example:
  - In words: Student's are actually getting less than the recommended amount of sleep
  - H_a: $\mu<8$

General Hypotheses for μ (3 types)

(μ_0 is your claimed value)

<u>Left-sided (tailed) Hypotheses</u>

H_0: $\mu = \mu_0$

H_a: $\mu < \mu_0$

<u>Right-sided (tailed) Hypotheses</u>

H_0: $\mu = \mu_0$

H_a: $\mu > \mu_0$

<u>Two-sided (tailed) Hypotheses</u>

H_0: $\mu = \mu_0$

H_a: $\mu \neq \mu_0$

Hypotheses Examples

- Is nicotine content greater than the written 1 mg/cigarette, on average?

$$H_0\text{: } \mu = 1$$
$$H_a\text{: } \mu > 1$$

- Does a drug create a change in average (systolic) blood pressure?

$$H_0\text{: } \mu = 120$$
$$H_a\text{: } \mu \neq 120$$

Results of A Hypothesis Test

- The null hypothesis (H_0) is assumed to be true throughout the statistical analysis.

- Two Results :
  - Only if the sample observations are in extreme contradiction to H_0 do we reject H_0 in favor of H_a.

  - If H_0 cannot be rejected, we do not conclude that H_0 is true (or accept it) but merely that we do not have evidence to reject it. Therefore we fail to reject H_0

What can really happen

In actuality there are Two possibilities:

1) H_0 is true, the difference between the sample proportion and the population proportion is due to chance.

2) H_0 is false, the sample came from a population whose proportion is not the same.

What can really happen

So there are actually four outcomes (two are correct).

A null hypothesis may or not be true, and a decision is made to reject or not reject it after running a hypothesis test.

	H_0 is true	H_0 is false
Reject H_0	Type I error	Correct
Don't reject H_0	Correct	Type II error

<u>Type I error</u> occurs if one rejects a true H_0

<u>Type II error</u> occurs if one does not reject H_0 when it is false.

Examples of errors

- Our justice system works by this rule: “Innocent until proven guilty”
  - H_0: defendant is innocent
  - H_a: defendant is not innocent

- But, Errors do occur:
  - Type I error – convicted when innocent
  - Type II error – acquitted when not innocent

- Think about this in medical tests
  - False + or false –

Quantifying Error

P(Type I error) = α. This is called the significance level.

- α is called the significance level.
- Typical depends values for α are 0.10, 0.05, and 0.01.

P(Type II error) = β.

- β Depends heavily on alpha and sample size
- Power of a test = $1 - \beta$

Relationship

- α and β are inversely related. Any attempt to reduce one will increase the other.
- The only way to reduce the probability of both types of errors is to increase the sample size.

Steps to solve a hypothesis test

1) State the appropriate hypotheses
   - Null (equality) and Alternative ($<, >, \neq$)
2) State the appropriate test statistic
3) State the Critical Region
4) Conduct the experiment and calculate the test statistic
5) Draw your conclusion

Test Statistic

- Measures compatibility between the null hypothesis and the data collected. It is used for the probability calculation needed for our hypothesis test.
- When the test statistic is far away from the value we would expect that if the null hypothesis is true, we reject the null hypothesis and conclude the evidence supports the alternative hypothesis.

Test Statistic: Hypothesis Test of the population mean (σ known)

- For a hypothesis test, the test statistic should have the structure:

$$\text{TS} = \frac{\text{obs} - \text{mean}}{SE}$$

- So if we know σ and CLT checks out, our test statistic should be:

$$z_0 = \frac{\bar{x} - \mu_o}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

Note: Sometimes denoted as z_0, z_t or just z

Steps to solve a hypothesis test

1) State the appropriate hypotheses
   - Null (equality) and Alternative ($<, >, \neq$)
2) State the appropriate test statistic
   - Based on the sampling distribution of our estimate
3) State the Critical Region
4) Conduct the experiment and calculate the test statistic
5) Draw your conclusion

The Critical Value Method

- While the p-value Method (more on this later) is preferred, It may be best to start by using the Critical Value method.

- The CV method consists of:
  - Finding a critical value based on α
  - Define your rejection region based on the type of test you have (left, right, two)
  - Asses where your test statistic falls.
    - If it falls in the rejection region, REJECT the Null
    - If it does not, the we FTR.

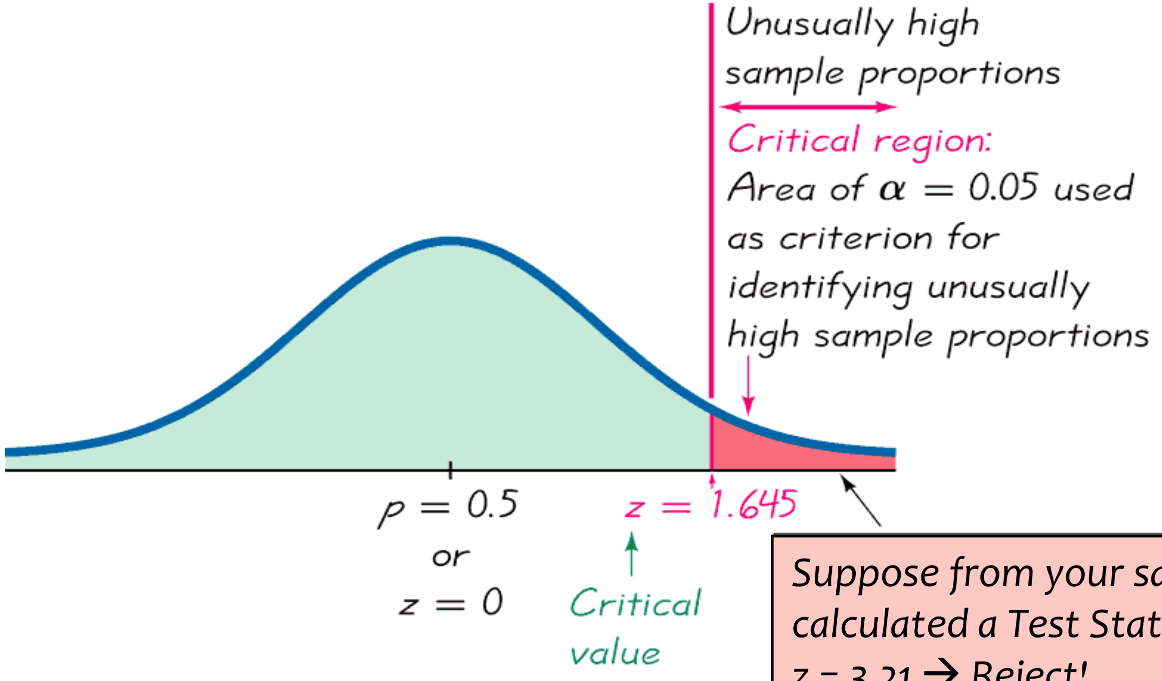
Finding A Critical Value (For a HT)

- Draw the figure and indicate the appropriate area.
  - If the test is left-tailed, the critical region, with an area equal to α, will be on the left side of the mean.
  - If the test is right-tailed, the critical region, with an area equal to α, will be on the right side of the mean.
  - If the test is two-tailed, α must be divided by 2; one-half of the area will be to the right of the mean, and one-half will be to the left of the mean.

- Find the z value in your table.
  - For a left-tailed test, use the z value that corresponds to the area equivalent to α.
  - For a right-tailed test, use the z value that corresponds to the area equivalent to $1 - \alpha$.
  - For a two-tailed test, use the z value that corresponds to $\alpha / 2$ for the left value. It will be negative. For the right value, use the z value that corresponds to the area equivalent to $1 - \alpha / 2$. It will be positive.

The Critical Value Method

The rejection region is the set of all values of the test statistic that cause us to reject the null hypothesis. For example, see the red-shaded region here based on a right tailed test w/ α=0.05.



Suppose from your sample you calculated a Test Statistic of:
$z = 3.21$ → Reject!

However, if your Test statistic was $z=1.24$ → FTR

Steps to solve a hypothesis test

1) State the appropriate hypotheses
   - Null (equality) and Alternative ($<, >, \neq$)
2) State the appropriate test statistic
   - Based on the sampling distribution of our estimate
3) State the Critical Region
   - Based on α
4) Conduct the experiment and calculate the test statistic
5) Draw your conclusion

Conduct the experiment and calculate the test statistic

- Once we collect the data we can calculate our test statistic, compare it to our C.V. and decide to reject or FTR.
- This can give us an answer to our question, but we can take it a step further…..

P–Value Approach

P–Value – The probability, computed assuming H_0 is true, that the test statistic would take a value as extreme or more extreme than that actually observed.

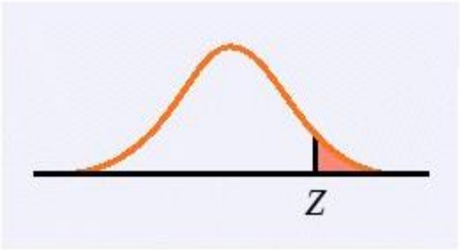
The smaller the *p*–value, the stronger the evidence against H_0. (The easier it will be to reject H_0).

You P–value is based on your Test Statistic and the type of test you are running

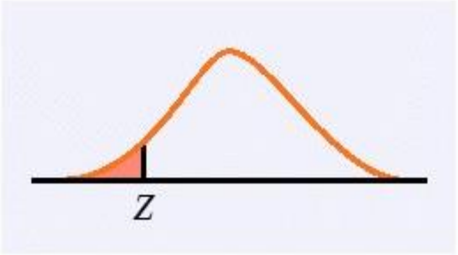
P-values and one or two sided hypotheses

First let's consider a Z test statistic. The lower case z is the test statistic value.

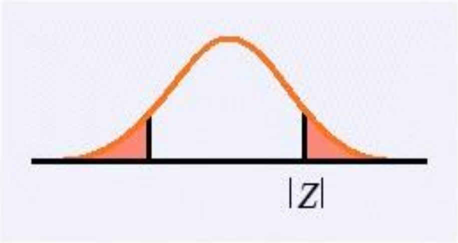
$H_a: p > p_0$ is $P(Z \geq z)$



$H_a: p < p_0$ is $P(Z \leq z)$



$H_a: p \neq p_0$ is $2P(Z \geq |z|)$



Measuring Surprise: The *p*-value

The null hypothesis tells us what to expect when we look at our data. If we see something unexpected – that is, when we are surprised – then we should doubt the null hypothesis. If we are *really* surprised, we should reject it altogether.

The p-value gives us a way to numerically measure our surprise. It reports the probability that, if the null hypothesis is true, our test statistic will have a value as extreme as or more extreme than the value we actually observe. Small *p*–values (close to 0) mean we are *really* surprised. Large *p*–values (close to 1) mean we are *not* surprised at all.

The significance level α

We compare the p–value with the significance level, α. This value is decided <u>before</u> conducting the test.

- If the p–value is less than or equal to α ($p \leq \alpha$), then we **reject H_0**.
  - We might say we have ***statistically significant*** evidence at level α
- If the p–value is greater than α ($p > \alpha$), then we **fail to reject H_0**.
  - We do not have sufficient evidence to reject in this case

Steps to solve a hypothesis test

1) State the appropriate hypotheses
   - Null (equality) and Alternative ($<, >, \neq$)
2) State the appropriate test statistic
   - Based on the sampling distribution of our estimate
3) State the Critical Region
   - Based on α
4) Conduct the experiment and calculate the test statistic
   - Also includes p-value method
5) Draw your conclusion

We will demonstrate these in the context of an example

Hypothesis Test Example (for μ, σ known)

The number of customers of a drive-thru bank has averaged 20 per hour, with $\sigma = 3$ per hour. The bank manager feels that new competition will reduce the number of customers. 36 randomly selected hours yielded a sample mean of 19.39. Test the manager's claim at $\alpha = .05$.

Solution

1) State the hypotheses.

Identify the claim:

(The mean # of customers will decrease)

H_0 is the "no change" hypothesis

H_0: $\mu = 20$

H_a: $\mu < 20$

We see that it is a left tailed test.

Full Solution

2) State the appropriate Tests Statistic:

- We see that our parameter of interest here is μ (mean number of customers)

- Is σ known?
  - We are given the population SD σ. No info about population, but n > 30.
  - Yes, we can use a Z test statistic:

$$z = \frac{\bar{x} - \mu}{\left(\frac{\sigma}{\sqrt{n}}\right)}$$

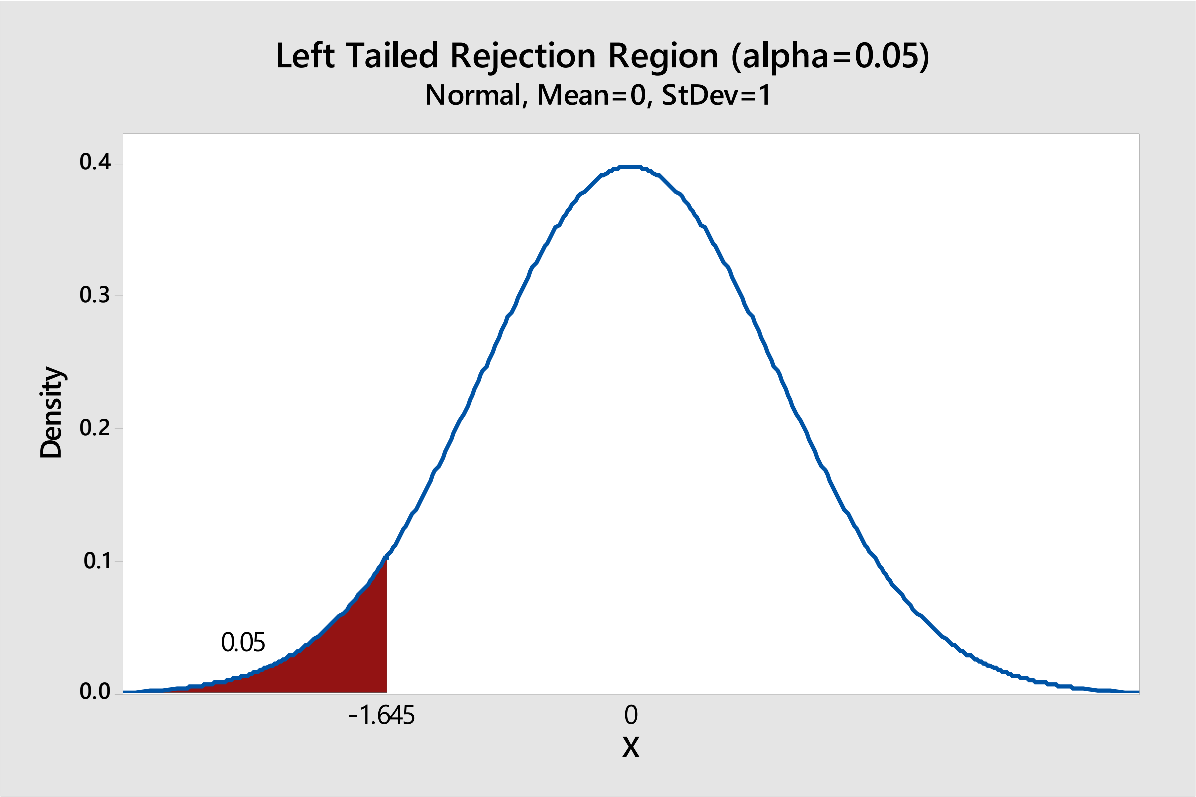
Full Solution

3) State the Critical Region

One tailed (left), $\alpha = .05$;

Look up critical value on standard normal curve

$z = -1.645$ is the critical value



Full Solution

4) Conduct the experiment and calculate the test statistic

Use the Z test statistic.

36 randomly selected hours yielded a sample mean of 19.39

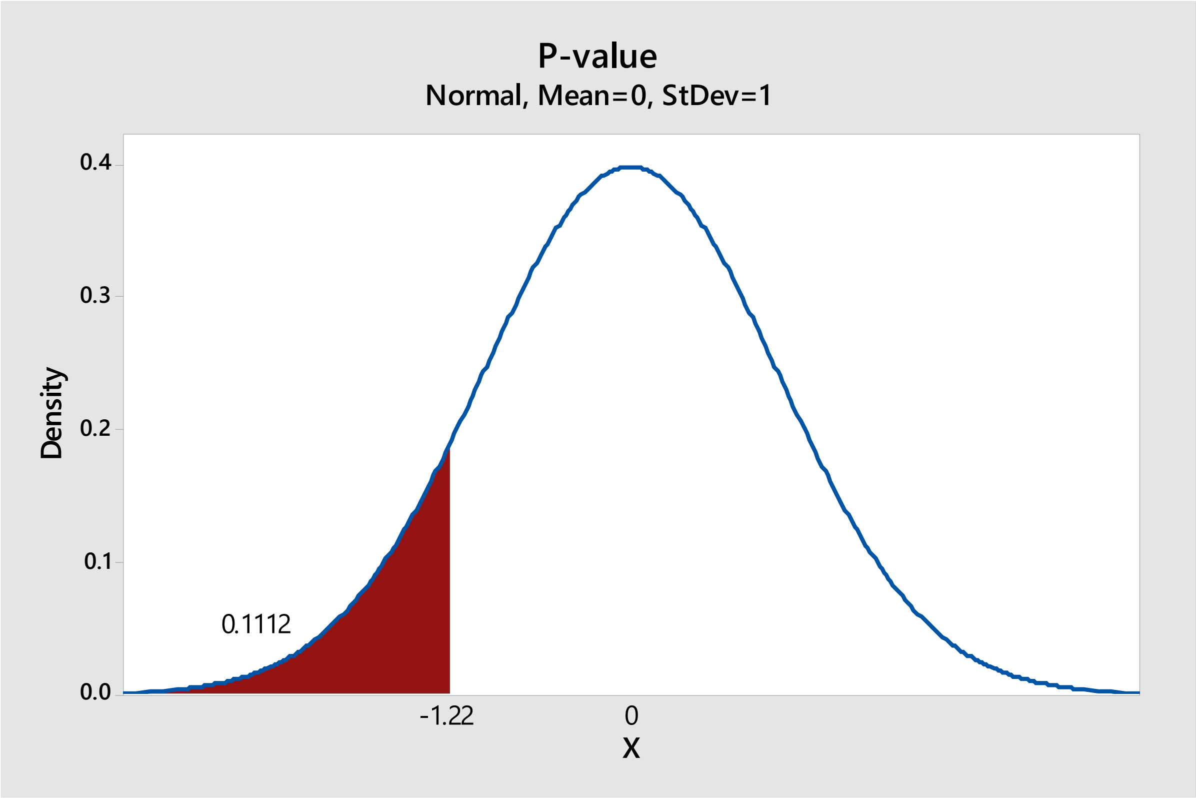
$$Z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{19.39 - 20}{3/\sqrt{36}} = \frac{-.61}{.50} = -1.22$$

Full Solution

4) Can Find P–value

Since $H_1: \mu < 20$ is one tailed use $P(Z < -1.22)$

Value from Z–table is 0.1112



Full Solution

5a) Draw Conclusion

<u>Critical Value Method</u>

Our Z test statistic of -1.22 does not fall in the rejection region

Thus, we cannot reject H_0

<u>P-Value Method</u>

Since we $\alpha = .05$, we can use it to compare

$0.1112 > .05$ hence we cannot reject H_0

We did not reject H_0 in either approach.

We do not have significant evidence to infer that H_0: $\mu = 20$ is false.

Since we are not rejecting H_0, we conclude that there is not enough (significant) evidence to infer that the alt. hypothesis H_a is true.

Full Solution

5b) Interpret results

Simply put our results in terms of the question

Is this sufficient evidence to indicate that the number of customers decreased with the new competition?

We do not have significant evidence (at $\alpha = .05$) that the mean number of customers has decreased.

Statistical Significance vs. Practical Significance

- Statistically significant findings do not necessarily mean the results are *useful*.

- Practically insignificant differences.
  - Ex. For $\alpha = 0.05$ a P-val=0.049 (reject) vs. P-val=0.051 (FTR)

- The departure from H_0 may be tiny & of little practical significance, particularly at large sample sizes.